Market Power and Price Informativeness*

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Abstract

Large institutional investors dominate asset ownership in financial markets worldwide. We develop a general equilibrium theory to study the distributional effects of asset ownership for price informativeness when some investors (oligopolists) have price impact and can learn about individual assets' payoffs. We find that price informativeness is non-monotonic in the oligopolists' sector size, decreasing in the sector's concentration, and in the size of the passive sector, the last effect arising jointly due to a decrease in aggregate information capacity and an amplification through an endogenous learning response. We decompose the transmission mechanism into (i) a learning channel and (ii) an information pass-through channel and show that the pass-through channel is a primary driver of the size results while both effects contribute to the concentration result.

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1 Introduction

Institutional investors play an important role in financial markets due to their economic size and the amount of information they produce. In 2019, their ownership of an average stock in the U.S. equaled around 60%. Within the institutional sector, active investors who produce information hold about the same share of the market as do passive investors who do not trade for information reasons. The ownership structure is heavily skewed, with the ten largest investors holding, on average, 35% of total shares outstanding, and it varies greatly across individual assets. The economic importance of institutional investors and their impact on market stability and welfare has attracted considerable attention from market participants, policy makers, and academics. One major consideration is the distributional effect of *large* active and passive investors on asset prices and, more broadly, on capital allocation efficiency. Both outcomes are economically important as they feed directly back to firms' costs of capital and aggregate welfare.

In this paper, we contribute to this discussion by theoretically analyzing the effects of changes in large investors' total size, the concentration of their ownership, and the extent of passive ownership on price informativeness, which we measure as the covariance of the price with the fundamental, normalized by the volatility of the price.[1] At the heart of our analysis lies an endogenous trade-off between information acquisition and trading decisions of investors of different sizes. On the one hand, large active investors have greater capacity to acquire private signals, the use of which could increase the amount of information revealed in asset prices. On the other hand, they also recognize their price impact, which makes them trade less on any information they acquire and feeds back to their learning choices. We show that this tradeoff gives rise to two channels that determine the behavior of price informativeness: *the learning channel*, which isolates the pure effect of the (endogenous) quality of private signals; and the *information pass-through channel*, which quantifies how much investors actually act on the private signals in their trading decisions. Further, the interaction of these two forces results in price informativeness that has a non-monotonic relationship with aggregate size of the large investors and a strong negative relationship with concentration. Additionally, these forces amplify the negative general equilibrium effect of the rise of passive investors on price informativeness.

[1] This measure, used for example in Bai, Philippon, and Savov (2016) and Farboodi, Matray, Veldkamp, and Venkateswaran (2020), has a strong economic appeal in that it can be derived as a welfare measure using Q-theory. It increases with the correlation between the price and the fundamental, and the volatility of the fundamental, as correlation is more meaningful when the unobserved variable is more volatile. Analytically, it represents the reduction in the variance of posterior beliefs when agents use price as a signal about fundamentals.

To explore the endogenous interaction between learning and trading, we build a new theory of large investors with varying sizes. Our model features a mass of atomless competitive fringe that has no price impact, and l large *oligopolists*–investors whose trades move prices.[2] Oligopolists internalize their own (heterogeneous) price impact but also other investors' endogenous learning and portfolio decisions across multiple assets, which are allowed to be heterogeneous in terms of the supply process and fundamental volatility. Each oligopolist is endowed with a capacity to collect information, which they can use to reduce uncertainty about asset payoffs via their own private signals. Fringe investors do not have any such information capacity but they are relevant from the perspective of the market size distribution. Oligopolists also receive a public signal about assets' payoffs from market prices. This feature allows us to capture explicitly the information revelation underlying the trades of informed oligopolists, and the ensuing strategic interaction with other oligopolists. We model individual learning choices using the theory of rational inattention of Sims (2003), where investors allocate their learning capacity optimally across assets, depending on the assets' characteristics and the investors' objective functions. After learning choices have been made (in a Nash equilibrium), trading takes place via demand schedule competition among oligopolists. Broadly, our theory extends the work of Kyle (1989) and Vives (2011) to allow for endogenous information choices under non-symmetric allocations of information and trades.[3] The equilibrium is a fixed point involving not only demand schedules but also learning choices across multiple assets and multiple oligopolists. Both choices are asymmetric if oligopolists differ in terms of their size. This generalization is non-trivial and allows us to address new questions concerning the impact of large investors' aggregate size and concentration on price informativeness as well as the role of differential information capacity when we include passive investors as a special case.

Given our definition of price informativeness, we can analytically decompose the impact of any market structure on the informational content of prices into the *the information pass-through channel*, which isolates the impact of oligopolists' trading responses to private signals, and the *the learning channel*, which isolates the pure effect of the quality of the oligopolists' signals holding trading decisions fixed. Each of the channels impacts price informativeness through the covariance of the price with the fundamental, and through the orthogonal noise in the price coming from

[2] This modeling assumption stands in contrast to that in the literature with oligopolistic traders and noise traders, in which oligopolists make up 100% of the market; hence, comparative statics with respect to the sector size are trivially ruled out. Additionally, the presence of competitive fringe ensures existence of equilibrium for small number of strategic traders, in contrast to Kyle (1989).

[3] Relative to a portfolio choice theory with endogenous information (Van Nieuwerburgh and Veldkamp (2009, 2010), and Kacperczyk, Van Nieuwerburgh, and Veldkamp (2016)), we introduce large investors and price impact.

errors in oligopolists' signals. The learning channel and the information pass-through channel enter the covariance and the noise together as complementarities, and hence require additional analysis to disentangle. Intuitively, the learning channel determines the quality of the oligopolists' signals, while the information pass-through channel determines how much of these signals actually result in trades and hence ultimately show up in prices. In the paper, we characterize the net effects and the impact of each of the channels analytically and numerically.

In order to gain further insights into the workings and interactions of the two channels, we consider several special cases of the model, in which we analytically characterize a set of comparative statics results. For those special cases, we rule out learning from prices and specialize the size distribution. First, we show that in the perfectly competitive limit case, increasing in the total size of informed investors increases the price informativeness for each asset. In this case, the learning and information pass-through channels collapse to just the learning channel which is strictly increasing in the size of informed investors. Intuitively, because investors have no price impact, they always fully act on their information, which then gets revealed in asset prices. With a larger mass of informed investors each asset receives more information capacity devoted to learning about its payoff. Hence, price informativeness increases both in the aggregate and in the cross-section.

Second, we examine the case of a monopolistic investor to highlight the effect of market power. We show that aggregate price informativeness is *non-monotonic* in the size of the monopolist, first increasing—just like in the competitive limit—but eventually decreasing to zero as the monopolist's size grows. The primary force responsible for this non-monotonic behavior is the information pass-through channel. Intuitively, as the price impact of the monopolist grows too much, it is optimal for him to actually reduce his trading sensitivity to his signals. The incentive to reduce trading on private information due to price impact also drives the monopolist to shift his learning choices and gradually expand the set of assets that he learns about as his size increases. If large size prevents the monopolist from trading on private information about an asset, then endogenously learning less about that asset and using the information capacity elsewhere is optimal. In the numerical section, we show that this endogenous learning adjustment operates in the general setting, and has an economically large impact on price informativeness.

The third case we consider is a duopoly setting with an exogenous information structure. Analyzing this case allows us to gain insights into the effects of strategic interactions between two large traders, on top of the pure size effects uncovered in the monopolistic setting. We show that from the perspective of maximizing price informativeness, the symmetric size distribution is only optimal under symmetric information structure–hence any deviation from symmetric information implies a deviation from symmetric optimal size distribution. This result implies that endogenizing learning choices can have profound implications for the conclusions about the impact of market structure on price informativeness—a result we confirm in the numerical section. The second insight from the duopoly analysis comes when we restrict one duopolist to be passive, that is, having zero information capacity. We show that in such a situation, the active duopolist lowers its information pass-through relative to the case in which it competes against another active duopolist. The mechanism works through the fact that competing against a passive duopolist in equilibrium results in a higher price impact of the active oligopolists, resulting in their lower holdings and lower information pass-through. This implies an additional reduction in price informativeness over and above that induced by a pure decrease in the relative size of the active oligopolist.

Given the model's analytical complexity, we rely on numerical solutions to study the predictions of the general benchmark model with endogenous information choice and learning from prices. In the first experiment, we increase the size of the entire oligopolistic sector and find a hump-shaped response of price informativeness, on an asset-by-asset basis and in the aggregate. A decomposition of price informativeness into the learning channel and the information pass-through channel reveals that the latter is the primary force driving informativeness. The results from this experiment corroborate our intuitions from the monopolistic case. In the second experiment, we change the concentration of the oligopolistic sector, while holding its total size unchanged. We show that an increase in concentration reduces price informativeness, driven both by the learning and the information pass-through channels. Intuitively, an increased concentration in this experiment means polarization of sizes and a smaller information pass-through for both the oligopolist that grows in size (because of the growing price impact) and the one that diminishes in size (because of a lower economic importance), the implication being a reduction in price informativeness.

In the last experiment, we increase the size of the passive sector at the cost of the active sector. We show that aggregate price informativeness generally decreases, yet still exhibits a hump-shaped pattern driven by information pass-through. A more nuanced picture emerges for individual assets. As the size of the passive sector increases, now relatively smaller active investors focus their learning on a smaller subset of assets. Specifically, assets with high returns to learning (assets with large supply in the model) observe an increase in their price informativeness, while assets in small supply, with lower returns to learning, observe a decrease. This heterogenous cross-sectional response to a growing passive sector is consistent with the empirical findings in Farboodi, Matray, Veldkamp, and Venkateswaran (2020).

Our final set of numerical results examines the role of endogenous learning. We compare the predictions of our benchmark model with a model in which learning choices are exogenously fixed, holding the capacity constraint the same. We find that fixing information choices significantly affects our conclusions. In particular, in the experiments with changing total size and the size of the passive sector, an exogenously fixed information choice pins down the size of the oligopolistic sector that maximizes price informativeness—the size can be significantly higher or lower than that of the benchmark model, depending on the choice of the exogenous information. For the concentration experiment, we show that fixing exogenous information can actually overturn the result that price informativeness is decreasing in concentration, giving instead a hump-shaped response. We conclude that allowing for endogenous learning is essential when studying the interaction of ownership structure and information content of asset prices.

1.1 Related Literature

The literature on informed trading with market power dates back to Kyle (1985) and Grinblatt and Ross (1985) whose setup is one strategic trader, and Kyle (1989) and Holden and Subrahmanyam (1992), which extend the model into an oligopolistic framework.[4] The effects of market size on price informativeness and efficiency have been studied in models of oligopolistic financial markets by Vives (2011), Vives (2014), and Rostek and Weretka (2012). These papers focus on symmetric strategies in a single asset environment, while we model investor and asset heterogeneity, both playing a crucial role for our results. Lambert, Ostrovsky, and Panov (2018) further extend the Kyle (1989) model to study the relation between the number of strategic traders and information content of prices in a general stochastic environment. In contrast to these studies, we model endogenous information choice, which we show is a key driver of our results. We also additionally examine the role of concentration and active/passive traders. Kyle, Ou-Yang, and Wei (2011) allow for endogenous information acquisition in a one-asset economy, but their mechanism involves differences in risk aversion. They focus on the contracting features of delegation. In turn, our framework features multiple assets and utilizes heterogeneity in size to study the information content of the price. Finally, Yang (2020) examines the role of price feedback effects in the oligopolistic market for firm disclosure and price informativeness.

[4] Models in which traders condition on others' decisions also include Foster and Viswanathan (1996) and Back, Cao, and Willard (2000).

Our general equilibrium model is anchored in the literature on endogenous information choice, in the spirit of Sims (1998, 2003). More closely related to our application are the models of costly information of Van Nieuwerburgh and Veldkamp (2009, 2010), Mondria (2010), Kacperczyk, Van Nieuwerburgh, and Veldkamp (2016), and Kacperczyk, Nosal, and Stevens (2019). Ours is the first theoretical study to introduce heterogeneous market power into a model with endogenous information acquisition. This novel aspect allows us to study strategic responses of oligopolistic traders in terms of their demand and information choices. Within the theory of rational inattention, we show that the framework with large investors leads to an interior learning solution, which is a contrast to the previous studies with competitive agents, in which each investor learns about one asset only. Finally, and very distinctly, the competitive framework would imply a strictly monotonic relationship between aggregate size and price informativeness. This relationship becomes hump shaped when price impact is explicitly modeled.

We also contribute to the literature on information production and asset prices. Bond, Edmans, and Goldstein (2012) survey the literature on information production in financial markets, emphasizing the differences between new information produced in markets (revelatory price efficiency: RPE) and what is already known and merely reflected in prices (forecasting price efficiency: FPE). Our focus is solely on RPE and is largely dictated by the modeling framework we use.[5] Goldstein and Yang (2015) study incentives for trading on private information and its effect on price informativeness, identifying effects similar to our measure information pass-through in a setup with exogenous information. Relative to that paper, we additionally introduce multiple assets and market power, as well as a public price signal. Stein (2009) develops a model of market efficiency and sophisticated (arbitrage) capital in the presence of capital constraints. Garleanu and Pedersen (2018) examine the role of search frictions in asset management for price efficiency. Breugem and Buss (2019) study the impact of benchmarking on price informativeness in a costly information acquisition competitive equilibrium model. Davila and Parlatore (2017) explore the equilibrium relation between price informativeness and price volatility, and characterize the conditions under which volatility and price informativeness co-move.

On an empirical front, Bai, Philippon, and Savov (2016) show that price informativeness is greater for stocks with greater institutional ownership. Our model delivers such a result for a range of ownership values. However, our theoretical analysis implies that, beyond certain levels,

[5] Theoretical work on asset prices and real efficiency also includes Dow and Gorton (1997), Subrahmanyam and Titman (1999), Kurlat and Veldkamp (2015), and Edmans, Goldstein, and Jiang (2015).

ownership may in fact reduce price informativeness, due to excessive price impact. Our micro-founded equilibrium model allows us to study the underlying economic mechanism in depth, as well as additionally investigate the role of ownership concentration and passive ownership. Farboodi, Matray, Veldkamp, and Venkateswaran (2020) examine differences in price informativeness between companies included and not included in the S&P 500 index. They show that the indexed companies exhibit larger efficiency and are the only ones to exhibit an increase in price informativeness, which they attribute to composition effect of these companies, being older and larger. We show that the predictions of our model are consistent with these empirical findings, suggesting that they may be partially due to a rise of passive investing. Kacperczyk, Sundaresan, and Wang (2020) show that the stock ownership by active institutional investors, domestic and foreign, causally increases price informativeness of stocks in which they invest more.

Finally, we add to a growing empirical literature that studies the impact of market structure in asset management on various economic outcomes. Following the diseconomies of scale argument of Chen, Hong, Huang, and Kubik (2004) and Pástor, Stambaugh, and Taylor (2015) show significant diseconomies of scale at the industry level. Using a merger between BlackRock and BGI as a shock to market power, Massa, Schumacher, and Yan (2020) study the asset allocation responses of their competitors. Our work complements these studies by studying theoretically the effect of ownership structure on price informativeness.

The rest of the paper is organized as follows. In Section 2, we present a set of motivating facts from the U.S. data on institutional ownership and its concentration. Section 3 and Section 4 present the theoretical framework, the equilibrium concept, and derive theoretical results for special cases of the model. In Section 5, we derive numerical solutions for the general model and discuss comparative statics. Section 6 concludes.

2 Motivating Facts

In this section, we present a number of facts that motivate our theoretical investigation. To position our model in the appropriate empirical context, we take institutional investors as a close empirical representation of investors with market power and private signals as modeled in our paper. Consequently, our motivating facts are based on a sample of such investors. We believe this assumption is not controversial given the extant evidence on the topic; however, we note that our model is not geared to confront other economically relevant aspects of institutional trading such as agency, optimal contracting, or managerial turnover.

First, we show that institutional investors hold a large fraction of equities in many developed economies. Second, we show that the holdings of the largest institutions, a measure of investor concentration, is very large in most markets. Finally, using U.S. market data, we demonstrate a large and increasing role of passive institutional investors. While we present time-series trends, our focus in the model is on the static effects of market structures observed in the data.[6]

Our results are derived from global institutional stock ownership data from Factset. Factset provides comprehensive information on institutional ownership of equity from over 70 countries. These data are considered the most comprehensive in the market and cover more than 98% of total value from publicly listed companies. The data are measured at a quarterly frequency and span the period 2000–2017.

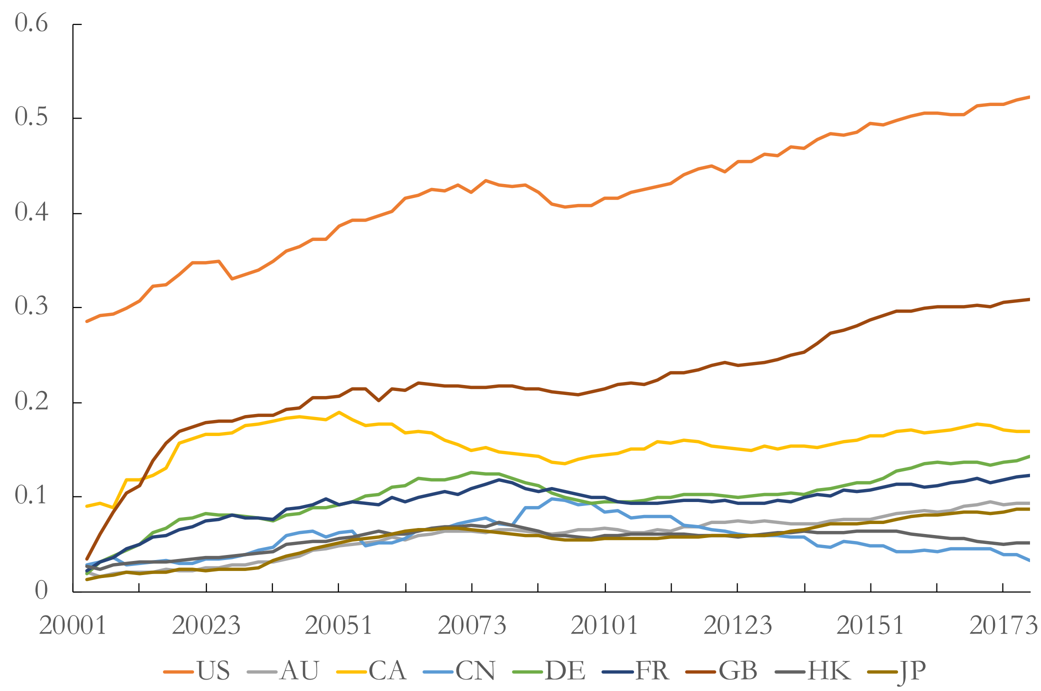

Figure 1: Institutional ownership worldwide.

Our first variable is institutional ownership, calculated as the stock-level ratio of the number of stocks held by financial institutions at the end of a given year to the number of shares outstanding. Since we are interested in market-level quantities, we aggregate measures by taking simple averages across all stocks in our sample. We use equal weighting, rather than value weighting, as this gives a

[6] A parallel microstructure literature (Boehmer and Kelley (2009)) examines empirically the relation between institutional ownership and price efficiency due to trading intensity. Efficiency there is measured using variance ratios and pricing errors. Their conclusions are akin to ours.

more conservative metric of the patterns in the data as institutions tend to favor large companies in their portfolios. We present the data for the largest equity markets worldwide, including Australia, Canada, China, France, Germany, Hong Kong, Japan, the United Kingdom, and the United States. We present the time-series evolution of institutional size in Figure 1. The data indicate a large cross-country variation in the importance of institutional owners. Capital market-based economies, such as the U.K. and the U.S., have the highest levels of institutional ownership, with the U.S. having an average ownership of almost 60%.[7] Bank-based economies, such as Germany, France, and Japan have lower levels of institutional holdings. However, except for China, all of these markets have witnessed a rapid increase in institutional ownership by 200% to 300% over the last 20 years. As institutional investors currently make up a significant percentage of total global asset holdings, questions of their optimal size are of increased importance.

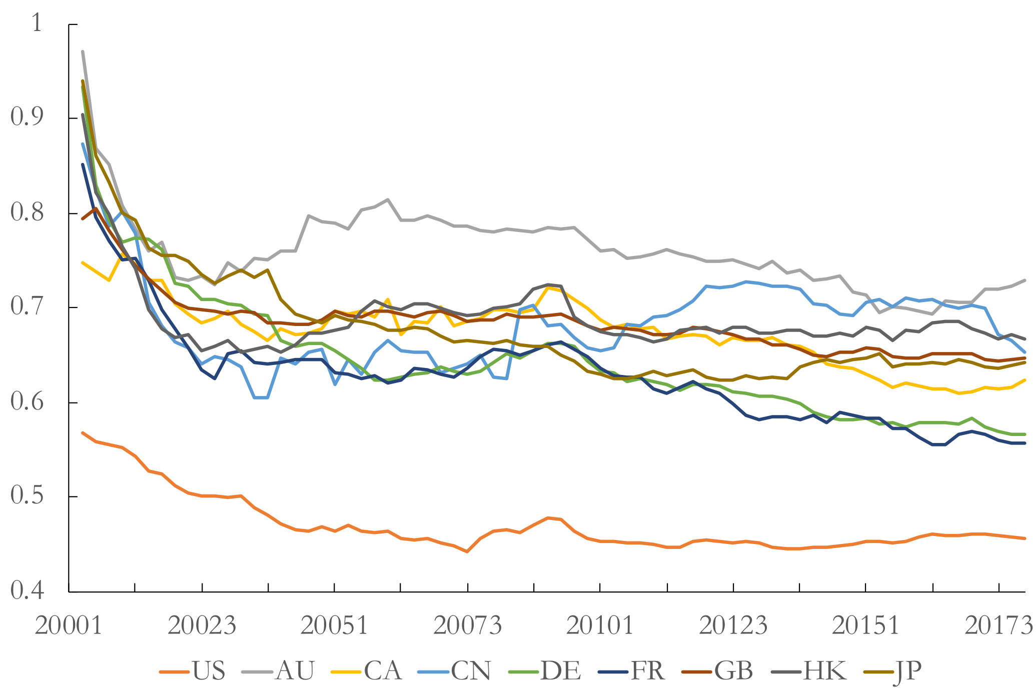

Figure 2: Top-5 investors as a share of total institutional ownership.

Next, we focus on market concentration. We define concentration as the ratio of the holdings of the top-5 largest institutions to total institutional holding for a given stock. As for ownership, we further average the shares across all stocks in each period. We present the time-series evolution of the country-specific quantities in Figure 2. Despite their different levels of institutional ownership, all the markets exhibit a high degree of market concentration, between 50% and 80%. Unlike the

[7]This number reaches almost 80% when we aggregate ownership using market weights of individual firms.

steady increase in the size of institutional investors over the past 20 years, the concentration levels have been stable over time. Of course, given the increase in total ownership of the sector, the largest players have increased their presence in the market, which makes their impact potentially much more significant.

Our final exercise relates to the distribution of ownership with respect to institutions' informational capacity. We define active investors as those engaged in information acquisition and passive investors as those who strictly invest in pre-defined index portfolios. The latter group includes both index mutual funds and ETFs. Since identifying passive funds in the data is not trivial, we use the evidence from the Investment Company Institute (ICI) Fact Book. This source restricts our analysis to the U.S. market alone, though we believe that similar conclusions are likely present in other markets as well. We calculate the share of passive ownership in total stock ownership of institutional investors and present the results for the period 1993–2017 in Figure 3. We observe a significant increase in passive ownership over time. While in mid–1990's passive funds accounted for less than 5% of the institutional equity market size in the U.S., this share has increased to 45% by 2018. In fact, as of 2019, passive funds manage more in equities than active funds do.

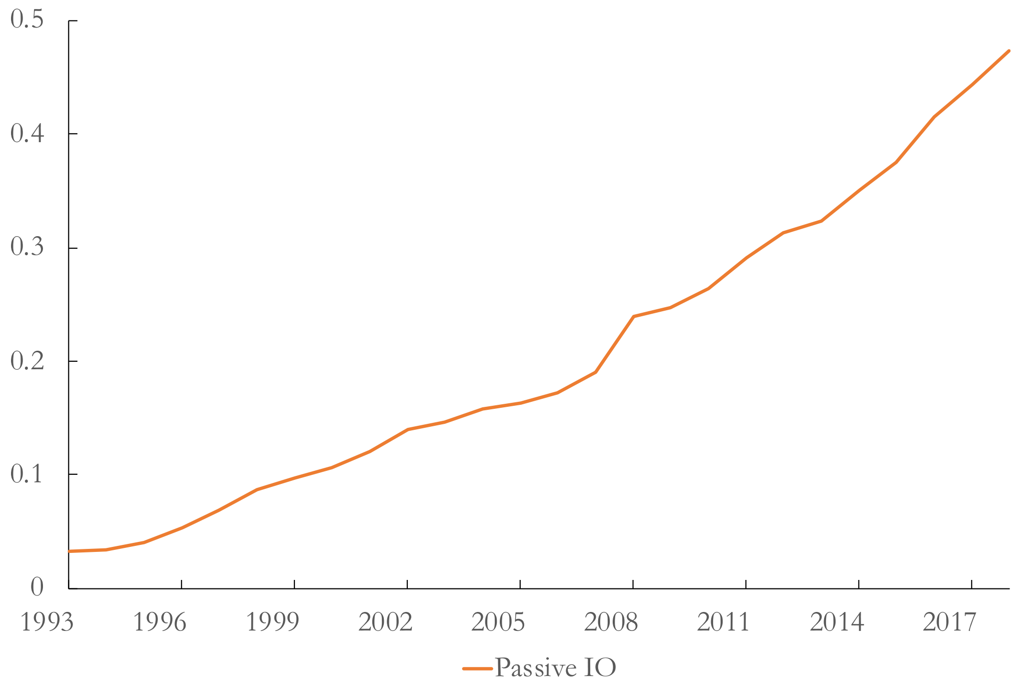

Figure 3: Passive ownership as a share of total institutional ownership.

3 Model

In this section, we set up a model of information and portfolio choices of investors who are constrained in their capacity to process information about asset payoffs. We allow for asset heterogeneity in terms of the supply process as well as fundamental volatility, while investors are heterogeneous with respect to their information capacity and size.[8] In particular, some of the investors are large in that they have price impact, which affects their optimization problem in both the information and portfolio choice dimensions.

Setup A continuum of investors of mass one is divided into two groups. One group, which we call the *competitive fringe*, is a mass λ_0 of atomless uninformed investors, indexed by h.[9] The second group consists of l *oligopolists*, indexed by j, each one having positive information capacity and size λ_j, such that $\sum_{j=1}^{l} \lambda_j = 1 - \lambda_0$. The sizes parameterized by λs map monotonically into ownership and hence we will use them as a proxy for ownership shares in our experiments. Oligopolists are large in the sense that they have positive mass, and hence have price impact, which they internalize when optimizing. Each investor solves an information capacity allocation problem and portfolio choice problem to maximize the expectation of mean-variance utility over end-of-period wealth (W_h or W_j) with common risk aversion coefficient, $\rho > 0$.[10]

The financial market consists of a risk-free asset, with price normalized to 1 and net payoff r, and $n > 1$ risky assets, indexed by i, with prices p_i and independent payoffs $z_i = \bar{z} + \varepsilon_i$, with $\varepsilon_i \sim N(0, \sigma_i^2)$.[11] The risk-free asset has unlimited supply, and each risky asset has a heterogeneous supply $\bar{x}_i$, which is subject to stochastic shocks $\nu_i \sim N(0, \sigma_{xi}^2)$, independent from payoffs and across assets, coming from non-optimizing noise traders, who trade for reasons orthogonal to prices and

[8]In the numerical section, we specifically focus on supply heterogeneity, but our analytical results apply more generally. As a robustness, we have also calculated results from a model with asset heterogeneity in the form of fundamental volatility, the factor that also provides a clear ranking of the returns from learning about an asset. The conclusions from the experiments are qualitatively similar.

[9]The presence of fringe investors allows us to naturally construct experiments in which we vary the size of the overall large investors sector share relative to the total. Crucially, it also the presence of competitive fringe ensures existence of equilibrium for small number of strategic traders, in contrast to Kyle (1989).

[10]The mean-variance utility assumption is standard in the literature and is known to provide tractability to the model, allowing for at least partial analytical characterization of equilibrium. As a result of this assumption, our model does not feature wealth effects, as that would require different utility specification. In terms of preference heterogeneity, our numerical solution can easily allow for risk aversion heterogeneity among investors–a dimension we do not pursue in this paper.

[11]Under the assumption of independence of signals across assets, assuming independent payoffs is without loss of generality, as asset payoffs can be easily orthogonalized under such assumptions. For a discussion, see Van Nieuwerburgh and Veldkamp (2010).

payoffs (e.g., liquidity, hedging, or life-cycle reasons).[12]

Investors know the distributions of the shocks, but not their realizations (ε_i, ν_i). Prior to making their portfolio decisions, oligopolistic investors can obtain information about some or all of the risky asset payoffs in the form of private signals and a public price signal. The quality of the private signals is constrained by each investor's capacity to process information, $K_j > 0$, which places a limit on the reduction of uncertainty about asset payoffs. The limit is expressed as entropy reduction (Shannon (1948)), following the work by Sims (2003). Higher capacity can be interpreted as having more resources to gather and process news about different assets, and it translates into signals that track the realized payoffs with higher precision. Investors choose how to allocate attention across the different assets. After observing their private signals, the oligopolists also observe and update their beliefs based on the price, which we assume does not require information capacity.[13] Prices adjust endogenously to clear markets. Oligopolists are strategic and directly reason through the market clearing equation in order to determine their own price impact as well as how much to update their beliefs from the price signal.

3.1 Competitive Fringe

The problem of the competitive fringe is standard. Given common prior beliefs $(\bar{z}, \sigma_i^2)$, and equilibrium prices p_i, for each i, each competitive investor h chooses portfolio holdings q_{hi} to maximize their expected utility from terminal wealth W_h, that is, they solve:

$$U_h = \max_{\{q_{hi}\}_{i=1}^n} E[W_h] - \frac{\rho}{2} V[W_h] \quad s.t. \quad W_h = (1+r)W_0 + \sum_{i=1}^{n} q_{hi}(z_i - rp_i) \tag{1}$$

where, without loss of generality, we normalize W_0 to zero. The solution to this problem yields optimal portfolio holdings, given by:

$$q_{hi} = \frac{\bar{z} - rp_i}{\rho \sigma_i^2}. \tag{2}$$

[12]In principle, for versions of our model with multiple oligopolists, the introduction of noise traders is not strictly necessary, as the noise in oligopolists' signals always leaves orthogonal noise in the price. By maintaining this standard assumption, however, we ensure that the monopolistic and competitive versions of our model do not have full revelation of information from the price.

[13]Since the focus of our paper is on the strategic interactions among oligopolists, for tractability, we abstract from the competitive fringe learning. That is, they do not update their beliefs based on the price.

3.2 Oligopolists

We assume (and verify as an equilibrium) that the portfolio strategy of each oligopolist j for each i takes the form of a linear demand schedule which depends on the private signal, s_{ji}, and the price p_i, (as in Kyle (1989)):[14]

$$q_{ji} = \beta_{0ji} + \beta_{1ji} s_{ij} - \beta_{2ji} r p_i. \tag{3}$$

Here, β_{1ji} measures the response of quantity demanded by oligopolist j in response to an update in their posterior belief about the mean asset payoff due to private signals.[15] The price coefficient β_{2ji} measures the elasticity of the quantity demanded with respect to the price movements.

Trading strategy Given the posterior beliefs of oligopolist j, $(\hat{\mu}_{ji}, \hat{\sigma}^2_{ji})$, each oligopolist chooses a trading strategy, summarized by $\{\beta_{0ji}, \beta_{1ji}, \beta_{2ji}\}_{i=1,\ldots,n}$, as a best response to the other oligopolists' trading strategies $\{\beta_{0ki}, \beta_{1ki}, \beta_{2ki}\}_{k\neq j, i=1,\ldots,n}$, conditional on other oligopolists' learning choices (characterized below) and competitive fringe trading choices q_{hi}. Hence, for every learning choice, the $\{\beta_{0ji}, \beta_{1ji}, \beta_{2ji}\}_{j=1,\ldots,l, i=1,\ldots,n}$ are a Nash equilibrium.

For the demand schedules submitted by oligopolists to be part of a Nash equilibrium, they must be consistent with utility maximization. Given posterior beliefs of an oligopolist j after observing the private signal and the price signal, given by $(\hat{\mu}_{ji}, \hat{\sigma}^2_{ji})$, utility maximization is:

$$U_j = \max_{\{q_{ji}\}_{i=1}^n} E[W_j] - \frac{\rho}{2} V[W_j] \quad s.t. \quad W_j = (1+r)W_0 + \sum_{i=1}^{n} q_{ji}(z_i - rp_i), \tag{4}$$

where the expectation and variance are conditional on the oligopolist j's information set. As before, we normalize initial wealth W_0 to zero. The solution to the above problem is given by:

$$q_{ji} = \frac{\hat{\mu}_{ji} - rp_i}{\rho \hat{\sigma}^2_{ji} + r \frac{dp_i}{dq_{ji}}}. \tag{5}$$

The demand of an oligopolist j for asset i, depends on the level of price impact the oligopolist j has on the market, captured by dp_i/dq_{ji}. Oligopolists internalize their price impact when making

[14] This is in line with the contributions Vives (2011), Vives (2014), and Kyle (1989). Here, additionally to those papers, we must allow for heterogeneous βs, because the oligopolists will generically have endogenously heterogeneous information. This flexible setup encompasses demand schedule competition if oligopolist j takes other oligopolists' βs as given and also Cournot competition if oligopolist j takes other oligopolists' qs as given. The Cournot case is equivalent to setting $\beta_{2ki} = 0$ in equation (6).

[15] As we show below, in our setup the posterior mean is equal to the signal s_{ji}.

their trading decisions, given (2) and (3), which implies, using market clearing:[16]

$$\frac{dp_i}{dq_{ji}} = \frac{\rho\sigma_i^2\lambda_j}{\lambda_0 r + \rho\sigma_i^2\sum_{k\neq j}\lambda_k r\beta_{2ki}}. \tag{6}$$

The impact that an oligopolist j has on the market price depends positively on their size λ_j. In turn, that impact depends negatively on the sizes of other oligopolists or the fringe, as well as on the elasticities of other oligopolists' quantities to the price, captured by β_{2ki}. Intuitively, an increase in an oligopolist j's quantity implies that the equilibrium price goes up by *less* if an increase in the price induces other oligopolists' quantities to drop by more (that is, if they have high β_{2ki}'s). In other words, if the other oligopolists' demand is very price-elastic, that makes oligopolist j's price impact smaller.

Private signals We assume that all oligopolists observe their own private signals and then the price, which they use to update their beliefs without using additional information capacity. The choice of the vector of private signals $s_j = (s_{j1}, ..., s_{jn})$ about the vector of payoffs $z = (z_1, ..., z_n)$ is subject to a capacity constraint $I(z; s_j) \leq K_j$, where $I(z; s_j)$ quantifies the reduction in entropy of the payoffs conditional on the signals (defined below). For analytical tractability, we assume that the signals s_{ji} are independent across assets and investors. In this case, the total quantity of information obtained by an investor based on private signals is the sum of quantities of information obtained for each individual asset, $I(z_i, s_{ji})$. We can think of the information problem as a decomposition of each payoff into the signal component s_{ji} and a residual component δ_{ji} that represents the information lost because of the investor's capacity constraint, i.e. $z_i = s_{ji} + \delta_{ji}$. If the signal and the residual are independent, then posterior beliefs are also normally distributed random variables. In particular, an investor j's posterior beliefs about asset i's payoff, after observing the private signals, are distributed according to

$$z_i | s_{ji} \sim \mathcal{N}(\xi_{ji}, \eta_{ji}^2),$$

where the posterior mean and variance are given by Bayes' rule:

$$\xi_{ji} = \bar{z} + \frac{cov(z_i, s_{ji})}{var(s_{ji})}(s_{ji} - \bar{z}),$$

[16] Detailed derivation can be found in Appendix A.2.

$$\eta_{ji}^2 = \sigma_i^2 - \frac{cov^2(z_i, s_{ji})}{var(s_{ji})},$$

and $\bar{z}$ stands for the signal's unconditional mean. Since a private signal's structure is such that $z_i = s_{ji} + \delta_{ji}$, with δ_{ji} being the data loss, the information contained in the signals is given by

$$I(z_i, s_{ji}) = \frac{1}{2} \log \left(\prod_{i=1}^{n} \alpha_{ji} \right),$$

which gives rise to the capacity constraint:

$$\prod_{i=1}^{n} \alpha_{ji} \leq e^{2K_j},$$

where $\alpha_{ji} \equiv \frac{\sigma_i^2}{\eta_{ji}^2}$ summarizes learning choices for each asset.

Price signal Oligopolists submit demand schedules that condition on the price p_i, which is equivalent to them observing the price explicitly. Therefore, as long as processing information contained in the price is costless, the oligopolists will use the observation of the price, and update their beliefs according to Bayes' rule:

$$\hat{\mu}_{ji} = \xi_{ji} + \frac{cov_j(z_i, p_i)}{var_j(p_i)} | s_{ji}(p_i - E_j[p_i | s_{ji}]),$$

$$\hat{\sigma}_{ji}^2 = \eta_{ji}^2 - \frac{cov_j^2(z_i, p_i)}{var_j(p_i)} | s_{ji}.$$

Note that the update is oligopolist-specific, because after observing the private signals, the covariance and variance of the price are oligopolist-specific due to the fact that each oligopolist faces a different residual supply for each asset, depending on their individual trading strategy. This is in contrast to a perfectly competitive setup, where the covariance and variance of the price would remain common across investors.

Given (5), the demand schedules choices of oligopolists, conditional on information choices $\{\alpha_{ji}\}_{i=1,\dots,n;j=1,\dots,l}$, can be summarized as a fixed point $\{\beta_{0ji}, \beta_{1ji}, \beta_{2ji}\}_{i=1,\dots,n;j=1,\dots,l}$ of the system:[17]

$$\beta_{0ji} = -\gamma_{ji}\Gamma_{ji} \tag{7}$$
$$\beta_{1ji} = \frac{(1-\gamma_{ji}\theta_{ji})}{\rho\hat{\sigma}_{ji}^2 + r\frac{dp_i}{dq_{ji}}}, \tag{8}$$
$$\beta_{2ji} = \frac{1-\gamma_{ji}/r}{\rho\hat{\sigma}_{ji}^2 + r\frac{dp_i}{dq_{ji}}}, \tag{9}$$

where

$$\Gamma_{ji} = -\frac{1}{\Delta_i}\bar{x}_i\frac{\rho\sigma_i^2}{\lambda_0} + \frac{1}{\Delta_i}\frac{\rho\sigma_i^2}{\lambda_0}\sum_{k=1}^{l}\lambda_k\beta_{0ki} + \frac{1}{\Delta_i}\frac{\rho\sigma_i^2}{\lambda_0}\sum_{k\neq j}\lambda_k\beta_{1ki}\frac{1}{\alpha_{ki}}\bar{z} + \frac{1}{\Delta_i}\bar{z}$$

$$\frac{dp_i}{dq_{ji}} = \frac{\rho\sigma_i^2\lambda_j}{\lambda_0 r + \rho\sigma_i^2\sum_{k\neq j}\lambda_k r\beta_{2ki}},$$

$$\theta_{ji} = \frac{1}{\Delta_i}\frac{\rho\sigma_i^2}{\lambda_0}\left[\lambda_k\beta_{1ji} + \sum_{k\neq j}\lambda_k\beta_{1ki}(1-1/\alpha_{ki})\right],$$

$$\gamma_{ji} = \frac{cov_j(z_i,p_i)}{var_j(p_i)},$$

and

$$\Delta_i = 1 + \frac{\rho\sigma_i^2}{\lambda_0}\sum_{j=1}^{l}\lambda_j\beta_{2ji}.$$

In contrast to the perfectly competitive case presented in Section 4.1 below, the indirect utility of an oligopolist is a nonlinear function of each oligopolist's choices and also depends on other oligopolists' choices. We present the details of the derivation of oligopolist utility and explicit maximization problem in Appendix A.4.

3.3 Equilibrium

Denote $\bar{\alpha} = \{\alpha_{ji}\}_{i=1,...,n;j=1,...,l}$ and $\bar{\alpha}_{-j} = \{\alpha_{ji}\}_{i=1,...,n;j=1,...,j-1,j+1,...,l}$. Let $\bar{\beta}(\bar{\alpha}) = \{\beta_{0ij},\beta_{1ij},\beta_{2ij}\}_{i=1,...,n;j=1,...,l}$ be a solution to (7)-(9) for a given information choice $\bar{\alpha}$.[18] For $i = 1,...,n$ and $j = 1,...,l$, an equilibrium consists of information and quantity choices of the fringe and oligopolists $\{q_{hi},\alpha_{ji},q_{ji}\}$, $\bar{\beta}(\bar{\alpha})$, and price p_i, such that

1. q_{hi} satisfies (2).

[17] Detailed derivations can be found in Appendix A.3.

[18] In principle may be more than one $\bar{\beta}$ solution to the fixed point (7)-(9). However, in our numerical examples, we always find a unique positive solution.

2. For every j, $\{\alpha_{ji}\}_{i=1,\ldots,n}$ maximizes $E_0 U_j$ in (A.4), given $\bar{\beta}(\bar{\alpha})$ and $\bar{\alpha}_{-j}$. That is, every oligopolist's information choice is a best response to the other oligopolists' information choices, while all the oligopolists internalize the effect of their information choices on the equilibrium behavior of everyone's quantities captured by βs and q_{hi}s.

3. $\bar{\beta}(\bar{\alpha})$ satisfies (7)-(9) for every feasible $\bar{\alpha}$. That is, given information choices and β of the other oligopolists, each oligopolist's quantity choices are optimal.

4. q_{ji} is given by (3) for all i and j.

5. For all realizations of shocks z_i and private signals s_{ji}, the price p_i clears the market for all i, i.e.

$$\lambda_0 q_h + \sum_{j=1}^{l} \lambda_j q_{ji} = x_i.$$

It is a known problem in the literature that allowing for asymmetric strategies–in our case for learning and trading–introduces a significant level of complexity to the model, precluding analytical characterization of equilibrium existence (as discussed, for example, in Lambert, Ostrovsky, and Panov (2018)). In our general setup, each oligopolist faces a different residual demand function that depends on other oligopolists' strategies, and chooses potentially different slopes of their demand schedules due to the fact that information is endogenously asymmetric as well. We are able to provide existence and uniqueness proofs for simplified versions of our model presented in Section 4, which are closer to those featured in the literature.[19] These proofs are provided in Appendix A.14. Additionally, all of the results of Section 5 are generated from numerical solutions of the model in which all of the optimality conditions are satisfied with a very high numerical precision.[20]

3.4 Price Informativeness

Following the work of Bai, Philippon, and Savov (2016), we define price informativeness as the covariance of the price with the fundamental shock, normalized by the variance of the price. Given

[19]The presence of the competitive fringe allows for equilibrium existence in our setup with a small number of strategic traders, in contrast to Kyle (1989).

[20]Our model involves two layers of strategic interactions among oligopolists, in terms of their learning and trading strategies. Hence, uniqueness of equilibrium allocations is not guaranteed in general. For the parameterization we use in the numerical section, we find the learning strategy and best responses by always starting the solution algorithm from the largest oligopolist. We find that the algorithm finds the same solution independent of the initial guess. We also find that the allocation changes smoothly as we change parameters, suggesting that we are focusing on a single equilibrium outcome.

this definition, price informativeness in our model can be expressed as

$$PI_i = \frac{cov(p_i, z_i)}{\sigma_{pi}} = \frac{\sigma_i \sum_{j=1}^{l} \omega_{ji} \frac{\alpha_{ji}-1}{\alpha_{ji}}}{\sqrt{\frac{\sigma_{xi}^2}{\sigma_i^2} + \left[\sum_{j=1}^{l} \omega_{ji} \frac{\alpha_{ji}-1}{\alpha_{ji}}\right]^2 + \sum_{j=1}^{l} \omega_{ij}^2 \frac{\alpha_{ji}-1}{\alpha_{ji}^2}}}, \tag{10}$$

where

$$\omega_{ji} \equiv \frac{\partial \lambda_j q_{ji}}{\partial s_{ji}} = \lambda_j \beta_{1ji}$$

is the responsiveness of an oligopolist's total demand for asset i to their private signal s_{ji}, which we term *the information pass-through.* The PI measure maps well to our theory as the square root of the reduction of the variance of posterior beliefs of a Bayesian agent who learns from the price.[21]

The endogenous terms ω_{ji} and α_{ji} enter expression (10) in an intuitive way. First, learning choices captured by the α_{ji} terms affect price informativeness to the extent that they impact the demand choices of the oligopolists, that is, if they pass through to quantities via the ω_{ji}s. If the oligopolists adjust their demand a lot in response to private signals, that is, they have high ω_{ji}s, price informativeness will increase due to higher covariance term $\omega_{ji}\frac{\alpha_{ji}-1}{\alpha_{ji}}$. At the same time, higher responsiveness of quantities to private signals means that any errors in the signals also show up in oligopolists' demand, which decreases price informativeness via the terms $\omega_{ij}^2 \frac{\alpha_{ji}-1}{\alpha_{ji}^2}$ in the denominator. These terms capture the noise in oligopolists signals. In the numerical section, we isolate the effect of changing ω_{ji}s on price informativeness and term it *the information pass-through channel.* Second, for a given information pass-through, price informativeness is affected by learning choices captured by α_{ji}s. They increase price informativeness by increasing the covariance of the price with the fundamental via the terms $\omega_{ji}\frac{\alpha_{ji}-1}{\alpha_{ji}}$, but also affect the noise in the price via the residual noise in private signals through the terms $\omega_{ij}^2 \frac{\alpha_{ji}-1}{\alpha_{ji}^2}$.[22] When we isolate the impact of learning choices on price informativeness, we refer to these effects as *the learning channel.*

4 Equilibrium Characterization

Below, we present an analytical characterization of the impact of different market structures on price informativeness. We first consider a perfectly competitive case (*Perfect Competition*), followed by the case with a single large investor (*Monopoly*), and then the case with two investors endowed with exogenous information (*Duopoly*). We show that in the perfectly competitive case,

[21] See, the Appendix A.6 for details.

[22] These terms are non-monotonic in α. As an example, for $\alpha < 2$, they are first increasing, then decreasing.

price informativeness grows monotonically with the size of informed investor sector. This contrasts with the monopolistic case, where we show that price informativeness in hump shaped in the size of the monopoly, which is driven by the information pass-through channel. In the duopoly setting, we explore the interaction of information and the optimal distribution of sizes across duopolists, which highlights the importance of endogenizing information choice when studying price informativeness. We present numerical results for the general oligopoly case in Section 5, where we show that the intuitions from the cases considered here hold in the general model as well.

4.1 Perfect Competition

In this section, we assume that all investors are perfectly competitive price takers, with fraction λ_1 having positive capacity $K > 0$, and fraction $\lambda_0 = 1 - \lambda_1$ having zero capacity. For this case, we also assume no learning from prices.

The informed fringe investors solve the standard portfolio allocation problem (1), given their posterior beliefs, which results in optimal portfolio holdings given by:

$$q_{hi} = \frac{\hat{\mu}_{hi} - rp_i}{\rho \hat{\sigma}_{hi}^2}, \tag{11}$$

where $\hat{\mu}_{hi}$ and $\hat{\sigma}_{hi}^2$ are the mean and variance of investor h's posterior beliefs after observing their private signals.

Given the optimal portfolio holdings as a function of posterior beliefs, the ex-ante optimal distribution of signals maximizes the ex-ante expected utility:[23]

$$E_0[U_h] = \frac{1}{2\rho} \sum_{i=1}^{n} \frac{E_{0h}\left(\hat{\mu}_{hi} - rp_i\right)^2}{\widehat{\sigma}_{hi}^2}, \tag{12}$$

where the choice of the vector of signals $s_h = (s_{h1}, ..., s_{hn})$ about the vector of payoffs $z = (z_1, ..., z_n)$ is subject to a capacity constraint $I(z; s_h) \leq K_h$.

Following Admati (1985), we conjecture and later verify that prices are

$$p_i = a_i + b_i \varepsilon_i - c_i \nu_i, \tag{13}$$

where coefficients a_i, b_i, c_i are determined in equilibrium. Summarizing learning choices by $\alpha_{hi} \equiv$

[23] This analysis follows Kacperczyk, Nosal, and Stevens (2019).

$\frac{\sigma_i^2}{\hat{\sigma}_{hi}^2}$, we can then express the maximization problem of investor h as[24]

$$\max \sum_{i=1}^{n} G_i \alpha_{hi} \tag{14}$$
$$\text{subject to}$$
$$\frac{1}{2}\sum_{i=1}^{n} \log(\alpha_{hi}) \leq K,$$

where

$$G_i \equiv \frac{(\bar{z} - ra_i)^2}{\sigma_i^2} + (1 - rb_i)^2 + r^2 c_i^2 \frac{\sigma_{xi}^2}{\sigma_i^2}. \tag{15}$$

The linear objective function and convex constraint implies that each competitive investor h specializes in learning only about one of the assets. For the remaining assets, that investor's holdings are determined by prior beliefs. In equilibrium, all assets that are learned about give the same gain G_i, and all other assets offer strictly lower gains. The equilibrium of the competitive economy can then be summarized by the mass of informed agents that learn about asset i, $m_i \geq 0, \forall i$.

The market clearing condition is of the form:

$$\bar{x}_i + \nu_i = \frac{1}{\rho\sigma_i^2}\left[\bar{z}(1 + \Phi_i) + \Phi_i \varepsilon_i - rp_i(1 + \Phi_i)\right]$$

where $\Phi_i \equiv m_i(e^{2K} - 1)$ and m_i is the fraction of investors who learn about asset i ($\sum_i m_i = \lambda_1$). Rewriting the above equation, we get the following pricing function:

$$p_i = \frac{\bar{z}(1 + \Phi_i) - \bar{x}_i \rho\sigma_i^2}{r(1 + \Phi_i)} + \frac{\Phi_i}{r(1 + \Phi_i)}\epsilon_i - \frac{\rho\sigma_i^2}{r(1 + \Phi_i)}\nu_i,$$

which defines the a_i, b_i, and c_i in (13). Further, the expression for G_i is of the form:

$$G_i = \frac{(\bar{z} - ra_i)^2}{\sigma_i^2} + (1 - rb_i)^2 + \frac{r^2 c_i^2 \sigma_{xi}^2}{\sigma_i^2} = \frac{\rho^2(\bar{x}_i^2 + \sigma_{xi}^2)\sigma_i^2 + 1}{(1 + \Phi_i)^2}.$$

In the equation, G_i depends negatively on the mass of agents learning about asset i, captured by Φ_i, giving rise to the standard result of strategic substitutability in learning.

Since in equilibrium, $G_i s$ are equalized among assets that are learned about, for any two assets

[24] For detailed derivations of (14) and (15), see Appendix A.1.

i, k such that $m_i > 0$ and $m_k > 0$, we have:

$$(1 + m_i(e^{2K} - 1))\frac{1}{\sqrt{1 + \rho^2(\bar{x}_i^2 + \sigma_{xi}^2)\sigma_i^2}} = (1 + m_k(e^{2K} - 1))\frac{1}{\sqrt{1 + \rho^2(\bar{x}_k^2 + \sigma_{xk}^2)\sigma_k^2}}, \quad (16)$$

and hence, $m_i > m_k$ if and only if $(\bar{x}_i^2 + \sigma_{xi}^2)\sigma_i^2 > (\bar{x}_k^2 + \sigma_{xk}^2)\sigma_k^2$. The last condition implies that investors have preference to learn about assets that are in large supply ($\bar{x}_i$) or are more volatile (σ_{xi}^2 or σ_i^2). Additionally, G_is depend on λ_0 only through Φ_i, and $dG_i/d\Phi_i < 0$. As a consequence, and given $\sum_i m_i = \lambda_1$, we have:

Proposition 1. *For all assets $i = 1, ..., n$, $\frac{dm_i}{d\lambda_1} \geq 0$, with strict inequality for assets that are learned about.*

If the number of assets that are learned about does not change as a consequence of the increased λ_1, the above proposition is a direct consequence of equation (16). In the case in which that number increases by an additional asset, indexed k, it has to be that, at the point of evaluation of $dm_i/d\lambda_1$, G_k was exactly equal to G_i for all i such that $m_i > 0$. Hence, we can apply (16) to those assets versus asset k as well, and use the same arguments as in the case of no change in the number of assets that are learned about.

Implications for price informativeness Price informativeness in this case is given by:

$$PI_i = \frac{b_i\sigma_i^2}{\sqrt{b_i^2\sigma_i^2 + c_i^2\sigma_{xi}^2}} = \frac{\Phi_i\sigma_i}{\sqrt{\Phi_i^2 + \rho^2\sigma_i^2\sigma_{xi}^2}},$$

and it is strictly monotonic in Φ_i, which, in turn, is monotonic in m_i. Hence, Proposition 1 implies:

Corollary 1. *$\frac{dPI_i}{d\lambda_1} \geq 0 \, \forall i$, with strict inequality if the asset is learned about ($m_i > 0$).*

In sum, under the competitive equilibrium model, the individual asset's price informativeness is a strictly monotonic function of the size of the informed competitive investor sector, λ_1.

4.2 Monopoly

Our second analytical case is that of a single large informed investor ($l = 1$). In this case, the price impact term simplifies to $\frac{dp_i}{dq_{ji}} = \frac{\rho\sigma_i^2\lambda_1}{r\lambda_0}$ and the β terms are $\beta_{0ji} = 0$, and $\beta_{1ji} = \beta_{2ji} \equiv \beta_i$, as there is no additional information in the price over and above the private signal of the monopolist (and hence $\gamma_{ji} = 0$). The monopolist's expected utility is given by:

$$U_1 = \frac{1}{2\rho} \sum_{i=1}^{n} \frac{\frac{L_i}{\lambda_0}\alpha_i + \lambda_0(\alpha_i - 1)}{\lambda_0 + 2\lambda_1\alpha_i}, \tag{17}$$

where we simplified the learning notation by dropping the index on α_i and summarizing the learning benefit by $L_i \equiv \rho^2(\bar{x}_i^2 + \sigma_{xi}^2)\sigma_i^2$. The marginal utility from increasing learning about asset i is:

$$\frac{\partial U}{\partial \alpha_i} = \frac{L_i + 1 - \lambda_1^2}{(\lambda_0 + 2\lambda_1\alpha_i)^2}.$$

Optimality then dictates that, for all i:

$$\frac{L_i + 1 - \lambda_1^2}{(\lambda_0 + 2\lambda_1\alpha_i)^2}\alpha_i \le \theta \tag{18}$$

where θ is the Lagrange multiplier on the capacity constraint $\sum_{i=1}^{n} \log(\alpha_i) = 2K$. The condition holds with equality whenever $\alpha_i > 1$.

Define the gain from learning about asset i as $M_i \equiv \frac{L_i+1-\lambda_1^2}{(\lambda_0+2\lambda_1\alpha_i)^2}\alpha_i$. M_i is increasing in L_i, and hence, just like the competitive investor, the monopolist has preference for learning about assets that are volatile (σ_i^2 or σ_{xi}^2) or are in large supply ($\bar{x}_i$). However, the monopolist has an incentive to learn about multiple assets, as long as their size and information capacity are large enough. Specifically, let the assets be sorted, such that $L_1 > L_2 > ... > L_n$. Then, the following holds:

Proposition 2. *(i) For every K, there exists $\underline{\lambda} > 0$, such that for all $\lambda_1 < \underline{\lambda}$, the monopolist learns only about one asset, $i = 1$.*

(ii) Let $\lambda_1 > 1/3$. Then M_i is strictly decreasing in α_i for all i, and there exists $\bar{K}$ such that, for $K > \bar{K}$, there exists a set of cutoffs $\lambda_1(N)$ increasing in N, such that for $\lambda_1 > \lambda_1(N)$, the monopolist learns about at least N assets with the highest L_is.

The proposition above states that for λ_1 large enough and if capacity is not too small, as the monopolist grows in size, their learning choice involves a growing number of assets, and eventually the monopolist learns about all assets. It is important to note that even though, from the learning perspective, the investor in (i) behaves like a competitive investor, the aggregate allocation is very different and actually $PI_i > 0$ only for asset 1.

Implications for price informativeness Price informativeness in the monopoly case is:

$$PI_i = \frac{cov(p_i, z_i)}{\sigma_{pi}} = \frac{\sigma_i \lambda_1 \beta_i \frac{\alpha_i - 1}{\alpha_i}}{\sqrt{\frac{\sigma_{xi}^2}{\sigma_i^2} + \left[\lambda_1 \beta_i \frac{\alpha_i - 1}{\alpha_i}\right]^2 + \lambda_1^2 \beta_i^2 \frac{\alpha_i - 1}{\alpha_i^2}}},$$

where the information pass-through term, $\lambda_1 \beta_i$, is

$$\lambda_1 \beta_i = \frac{1}{\rho \sigma_i^2} \frac{\lambda_1 (1 - \lambda_1) \alpha_i}{1 - \lambda_1 + \lambda_1 \alpha_i}.$$

In equilibrium, the information pass-through is always non-negative, but non-monotonic, increasing for small values of λ_1, then eventually decreasing. The following proposition summarizes this result:

Proposition 3. *There exist λ_L and λ_H such that information pass-through is strictly increasing in λ_1 for $\lambda_1 < \lambda_L$, and strictly decreasing for $\lambda_1 > \lambda_H$. It is always non-negative and equal to zero for $\lambda_1 \in \{0, 1\}$.*

The implication of Proposition 3 is that the non-monotonicity of the information pass-through is going to contribute to the potential non-monotonicity of price informativeness on an asset-by-asset level, and quantitatively it can contribute to the non-monotonicity of the average price informativeness. Propositions 2 and 3 imply that for small λ_1, information pass-through is the only determinant of the shape of price informativeness, while Proposition 2 implies that information pass-through is the main determinant of the shape of price informativeness as λ_1 approaches 1. In fact, price informativeness converges to zero in that case, while learning still remains positive. That is because as λ_1 approaches 1, information pass-through approaches 0. Corollary 2 formalizes this result. This is in contrast to the perfectly competitive result of Corollary 1, where we show that PI is monotonic in the size of the informed investor sector.

Corollary 2. *Price informativeness is strictly increasing in λ for $\lambda < \lambda_L$, and strictly decreasing for $\lambda_1 > \lambda_H$. It is non-negative for all λ_1 and zero for $\lambda_1 \in \{0, 1\}$.*

In equilibrium, the remaining factor that affects price informativeness is the equilibrium adjustment of learning α_i, which can also be non-monotonic. The relative contribution of these two channels outside of the $[0, \lambda_L] \cup [\lambda_H, 1]$ set depends on the specifics of the parameters of the model. We study these effects in a general setup in Section 5.

4.3 Duopoly

We now turn to a setting with two large investors, j and k. Specifically, we study the impact of the investors' sizes, λ_j and λ_k, with $\lambda_j + \lambda_k = 1 - \lambda_0$, on their optimal choices of demand schedules, summarized by β_{1ji} and β_{1ki}, in a special case of our model with exogenous information choices, α_{ji}, α_{ki}, and no learning from prices.[25] Under these assumptions, $\beta_{2ji} = \beta_{1ji}$ and $\beta_{0ji} = 0$; henceforth, we use the notation β_{ji} for any non-zero β.[26] Despite the additional assumptions, it is useful to characterize the choice of demand schedules and its relation to information pass-through and price informativeness, as it helps us understand the additional role of asymmetric size and arbitrary information structures in a standard model with exogenous information (e.g. Kyle (1989)). In particular, we can isolate the effects of a passive investor's growing size on an active investor's information pass-through.

Our first result, summarized in Proposition 4, shows that, holding information and the oligopolists' total size constant, an oligopolist's own β_{ji} is negatively related to their own size and positively related to the other oligopolist's size, thus creating a force for lower information pass-through ($\omega_{ji} \equiv \lambda_j \beta_{ji}$) for larger oligopolists. The overall effect on information pass-through is ambiguous and depends on the size of the entire oligopolistic sector $\sum_j \lambda_j$ relative to the fringe, but the negative relation between β_{ji}s and individual sizes are the only force responsible for a hump-shaped information pass-through, just like in the monopoly case of the previous section.[27]

Proposition 4. $\frac{d\beta_{ji}}{d\lambda_j} < 0$ *and* $\frac{d\beta_{ji}}{d\lambda_k} > 0$.

Next, we show that in order to maximize price informativeness, the optimal size distribution of investors is a function of their individual learning decisions. Specifically, if learning is symmetric, then the optimal size distribution is also symmetric. If learning is asymmetric, the optimal size distribution is also asymmetric. Finally, if one agent is informed, and the other agent is not, it is not necessarily optimal, from the price informativeness perspective, to make the uninformed agent atomistic. These results constitute the essence of Proposition 5.

Proposition 5. *Define* $\lambda_1^*, \lambda_2^* = argmax_{\lambda_1,\lambda_2} PI_i$ *for given* α_{1i} *and* α_{2i}. *Then:*

1. *If* $\alpha_{1i} = \alpha_{2i}$, *then* $\lambda_1^* = \lambda_2^*$

[25] For the monopoly case, we do not need to make that assumption as the price reveals no additional information over and above the monopolistic signal.

[26] For the derivation of the solution to β_{ji}s, see Section A.10.

[27] We consider the full model under duopoly in the numerical section, in which we show that for the parametrized size of the oligopolistic sector, information pass-through is hump-shaped.

2. *If* $\alpha_{1i} \neq \alpha_{2i}$*, then* $\lambda_1^* \neq \lambda_2^*$

3. *If* $\alpha_{1i} = 1$*, then* $0 < \lambda_2^* \leq 1 - \lambda_0$

The first two results are important because of their implications for the broad theoretical literature on the topic. Previous studies modeling investment decisions of multiple informed oligopolists have dealt with the complexity of the model by assuming symmetry in sizes and information quality of those agents. Proposition 5 shows that such an assumption also implies that price informativeness is maximized at that point. However, in general, the optimal ownership structure depends on the (not necessarily symmetric) information choices of the participants. The last result of the proposition may be counterintuitive—one might expect that if one agent is informed and the other is uninformed, then naturally the informed agent should be as large as possible, while the uninformed should be as small as possible. However, this intuition ignores general equilibrium forces. Recall that in the monopoly setting, if the monopolist is as large as possible (the whole market), they would not trade, leading to zero price informativeness. Similarly here, making the informed agent larger could hurt price informativeness by causing their information pass-through to decrease.

The role of passive investors We can use the duopoly case to illustrate the effects of a growing size of passive investors on information revelation. We show that the information pass-through of a large active investor is always *strictly lower* if they face a large passive competitor, rather than another large active investor. This finding illustrates an additional amplification of the negative effect of a passive investor's growing size on price informativeness, *ceteris paribus*, and directly follows from Proposition 6 below. Intuitively, lower information quality of the large competitor increases the price impact of the oligopolist and hence reduces the benefit of acting on their private information. Of course, in equilibrium, such a response would also trigger an endogenous response of the agents' learning decisions. Hence, the precise magnitude of the effect is hard to capture analytically. We present the full effect numerically in Section 5.

Proposition 6. $\frac{d\beta_{ji}\lambda_j}{d\hat{\sigma}_{ki}^2} < 0$. *If one duopolist exogenously experiences a reduction in their information quality, then another duopolist's information pass-through is reduced.*

5 Numerical Analysis

The analytical results in Section 4 provide a clean way to understand the economic forces driving our results. However, since the theory we develop is fairly complex, we are not able to derive all the results in the general benchmark model. This limitation may be important for the robustness of some results, especially those involving endogenous learning choices with the oligopolistic structure. To address this potential concern, in this section, we generalize the oligopolistic setting to a full model that is free of any of the simplifying assumptions. We provide a set of numerical results from the solution to the equilibrium of the model[28] in order to document the response of price informativeness to changing size and concentration of the oligopolistic sector, as well as the growth of passive versus active sector. Our numerical results below are consistent and confirm the intuitions developed in Section 4.

In our simulations, we choose parameters with two goals in mind: they have to give statistics that are empirically relevant and the solution needs to involve some degree of learning. Specifically, we consider parameters such that the benchmark model exhibits: (i) learning about all assets for at least some size distributions, (ii) aggregate oligopoly holdings of between 50% and 70%, and (iii) market excess real return of around 7% (which corresponds to the average over the period 1980–2015). These targets pin down the size λ_0 (average across experiments if it changes), the risk aversion coefficient ρ, and the common capacity K_j. The risk-free rate is set to match 2.5% real return on 3-month T-bills.

The rest of the parameters do not have empirical targets, so we set them arbitrarily and verify the robustness of our results to changing them. Specifically, we select parameter values for the return distribution $\bar{z} = 10$ and $\sigma_i = 1.35$ for all i, the number of assets to $n = 10$, and the number of oligopolists to $l = 2$.[29] Risky assets are heterogeneous in their supply size, $\bar{x}_i$, which we interpolate linearly between 1 and 7, and σ_{xi}^2, for which we target a coefficient of variation of 0.2 for all i.[30] We parameterize the λ_0 and $\{\lambda_j\}$s according to each experiment below. The simulation generates equilibrium levels of price informativeness, oligopoly holdings, and oligopoly concentration for each asset. In our experiments, we use the λs as proxy for stock ownership. While these are not the same, the λ shares map monotonically into ownership shares and hence give basis for us interpreting changes of λs as changes in ownership. We report the parameter values in Table 1.

Below, we report results on the effects of different market structures on aggregate price infor-

[28] This approach involves solving a fixed point of the best responses of the oligopolists to each other's learning and trading policies.

[29] The choice is largely dictated by the computational tractability. $l = 2$ is the minimum number of oligopolists which allows us to speak about the effects of size, concentration, and passive ownership on price informativeness. Experiments with greater values of l do not alter our conclusions but limit the range of sizes we can consider.

[30] We have also studied asset heterogeneity in terms of payoff volatility, with no significant difference in the results.

Table 1: Parameter values.

Parameter	Symbol	Value
Mean payoff	$\bar{z}_i$	10
Supply	$\bar{x}_i$	$\in [1, 7]$, linear distribution across i
Number of assets	n, l	$10, 2$
Risk-free rate	$r - 1$	2.5%
Vol. of noise shocks	σ_{xi}	target coefficient of variation of 0.2 for all i
Vol. of asset payoffs	σ_i	1.35 for all i
Risk aversion	ρ	0.97
Information capacities	K_j	4 for all j except growth in passive investing experiment
Investor masses	λ_0, λ_j	depending on experiment

mativeness. In our exercises, each point corresponds to one solution of the model. We present the average and the cross-section of price informativeness, as well as the decomposition into the *learning channel* implied by changing α_{ji}s and the *information pass-through channel* implied by changing ω_{ji}s, as implied by equation (10). It is important to note that in all of the experiments in this section, we do not change the aggregate amount of information in the economy, as the maximum quality and the number of signals that investors receive does not change. All of the documented effects come solely due to the fact that the size of the agents choosing the signals and trades varies.[31]

5.1 Size of the Oligopoly Sector

In our first experiment, we examine how price informativeness changes in response to different levels of the total size of the oligopoly sector, $\sum_j \lambda_j \equiv 1 - \lambda_0$. Specifically, holding the relative distribution of λ_js fixed so that the larger oligopolist is six times larger than the smaller one, we solve the model with respect to different values of $\sum_j \lambda_j \equiv 1 - \lambda_0$, ranging from 20% to 95%. The experiment we conduct here can inform several types of regulation, such as limits on entry, limits on a per-agent size in a given market, or regulations which impose a size-dependent cost structure.

Figure 4 presents the relationship between the size of the oligopoly sector and aggregate price informativeness, while Figure 5 presents price informativeness on an asset-by-asset basis. Price informativeness exhibits a hump-shaped relationship with the parametrized size, which points to an interior solution for optimal oligopoly sector size from the perspective of maximizing price

[31]This is in contrast to the case of changing the mass of a continuum of informed investors, as in the competitive case, in which case the aggregate amount of information capacity in the economy changes.

informativeness. As the size of the sector increases, oligopolists endogenously adjust both their learning choices, captured by the learning channel α_{ji}, as well as their choices of how much to trade on their information, captured by the information pass-through channel ω_{ji}. Figure 4 decomposes the aggregate change into the contribution of the learning channel and of the information pass-through channel. The information pass-through channel is responsible for the humped shape of the price informativeness response to a changing size. When oligopolists are small relative to the fringe, increasing their size has a positive impact on their information pass-through—the increasing λ_j effect outweighs the decreasing β_{1ji} effect. However, above a certain size level, an additional increase in size actually has a negative effect on price informativeness—oligopolists lower their β_{1ji}s more than one-to-one with the size increase, acting *less* on their signals. Intuitively, their size is so large that if they trade on their signals, they move the price too much and hence lose the benefits of private information. Hence, when we fix ω_{ji}s at their values corresponding to a large size of the sector, price informativeness is uniformly lower, and the hump shape disappears. This result confirms the intuition developed in Section 4.2, Propositions 2 and 3, and Corollary 2.

The effect due to the learning channel is more muted but it is still quantitatively significant. The effects of endogenous information choice are a combination of two forces in the model: (i) incentives for diversification of learning, which benefit price informativeness and grow with sector size (and can be seen in Figure 5), and (ii) strategic substitutability in learning, which gets stronger with the growing size of the oligopolistic competitors. On the one hand, when the sector's size is small, learning is not diversified and the average PI suffers from the fact that for many assets (with small supply) information is not produced at all. On the other hand, as the sector becomes sufficiently large, strategic substitutability dictates that oligopolists learn about different subsets of assets, and hence on a per-asset basis, price informativeness is reduced. Consequently, when we fix αs at the values corresponding to a large sector size in Figure 4, the strategic substitutability effect lowers price informativeness for all sizes by up to 20%.[32] In conclusion, from the perspective of maximizing the information content of the price, both the information pass-through channel and the learning channel favor a medium-size oligopolistic sector.

The intuition laid out above also plays out on an asset-by-asset level, as shown in Figure 5. For high-supply assets (e.g., assets 8, 9, 10), we can clearly see the hump shape of price informativeness driven by information pass-through. For low-supply assets (e.g., assets $1-4$), we see the extensive

[32] If we plotted the counterfactual price informativeness for αs corresponding to a small size of the sector, price informativeness would also be strictly below the benchmark line, this time due to the diversification effects.

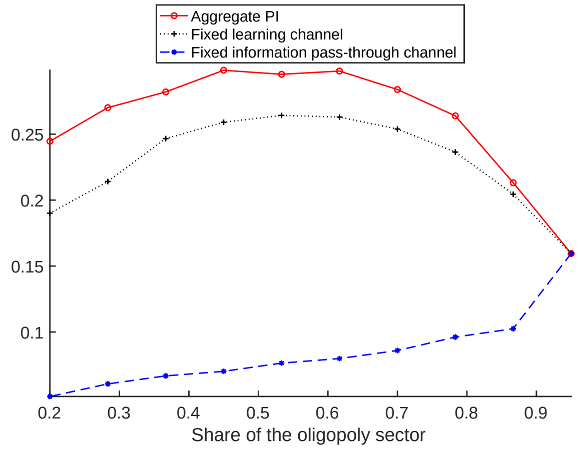

Figure 4: Decomposition of price informativeness and the oligopoly sector size $\sum_j \lambda_j$.

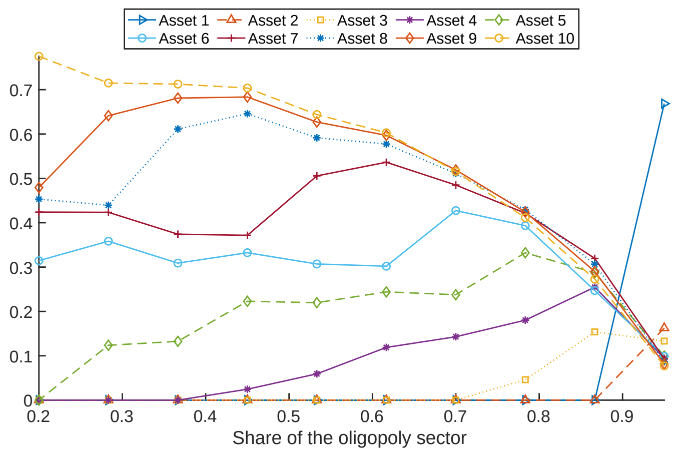

Figure 5: Asset-level price informativeness and the oligopoly sector size $\sum_j \lambda_j$.

margin of α_{ji}s matters more. As those assets enter the pool of assets learned about, they will contribute to the growth of average price informativeness. As some oligopolists start learning about those assets, they necessarily devote less capacity to the high-supply assets, and hence exacerbate the drop in those assets' price informativeness.

5.2 Concentration of the Oligopoly Sector

In our second experiment, we study the consequences of a change in the concentration of actively trading oligopoly sector for price informativeness. Specifically, holding the size of the oligopoly sector, $\sum_j \lambda_j$, constant at 55%, we vary the ratio of λ_1 to $\lambda_1 + \lambda_2$ to be between 52% and 92%. Figure 6 presents the aggregate PI, as well as its decomposition in which we keep the learning channel and the information pass-through channel fixed at the levels corresponding to the highest-concentration scenario.

We observe that price informativeness is decreasing in the oligopoly sector's concentration. The decomposition presented in Figure 6 is consistent with the intuition we developed in Section 3.4 and Section 4.2. As for the size experiment, the quantitative impact of the information pass-through channel is the most dominant. A high level of concentration leads to a lower information pass-through for *both* the large and small-size oligopolists, a result coming from its hump shape (following the intuition of the previous section and Corollary 2). Hence, the price informativeness levels reflecting a fixed information pass-through lie on a much lower, almost flat line. The contribution of the learning channel to the overall aggregate result is also quantitatively significant, if somewhat smaller than the information pass-through channel. Specifically, fixing the learning channel at the high concentration level significantly lowers the price informativeness curve relative to the benchmark experiment, implying a drop that is 50% smaller. This is because, with high concentration, smaller oligopolists specialize their learning on a narrower subset of assets, while larger oligopolists diversify their learning more. On net, this effect results in a reduction of average price informativeness.

In Figure 7, we additionally present the overall impact of concentration on price informativeness on an asset-by-asset basis. We can see that within our parametrization, the biggest contribution to the overall effect comes from price informativeness decreasing for all assets on the intensive margin rather than from the composition of actively traded assets. This is because the extensive margin in the aggregate is determined mostly by the total size of the oligopolistic market–a margin we keep constant in this experiment.

5.3 Growth in Passive Investing

The importance of oligopolistic traders results from two sources: their informational advantage and their size. While all oligopolists exert price impact, not all of them necessarily use informationally intensive trading strategies. To explore this tradeoff in more detail, in this section, we introduce a large passive investor, who does not produce her own information about asset payoffs.[33]

[33] We model passive investors as price-sensitive agents (they learn from prices) who do not have any information capacity of their own, which is consistent with Grossman and Stiglitz (1980). There are other ways to model or define passive investors, but the two characteristics that we believe are consistent across definitions are that passive investors

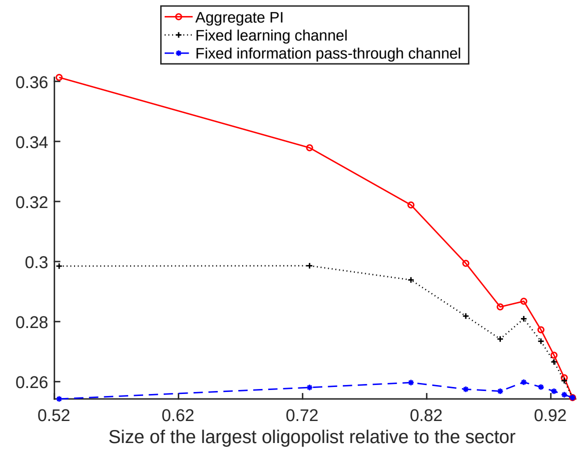

Figure 6: Price informativeness decomposition and the size of the largest oligopolist relative size $\frac{\lambda_1}{\lambda_1+\lambda_2}$.

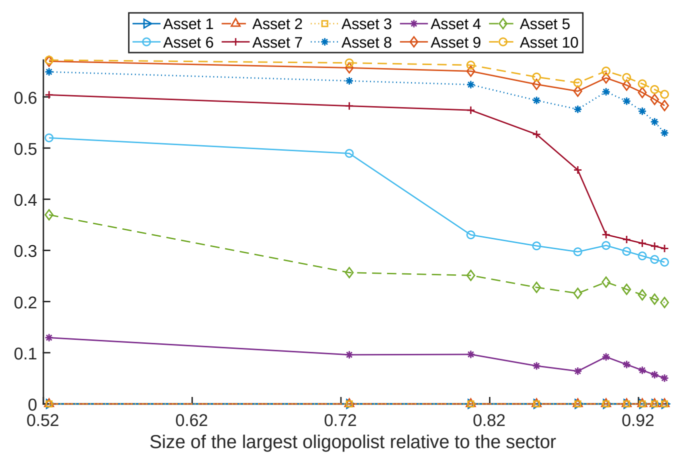

Figure 7: Asset-level price informativeness and the size of the largest oligopolist relative size $\frac{\lambda_1}{\lambda_1+\lambda_2}$.

We explore the predictions for price informativeness resulting from a growing size of the passive investor relative to that of the active investor. Specifically, we set $K_1 = 0$ and $K_2 = 7$ and vary the size of the passive investor relative to the active investor's from 0.01 to 10, while keeping the sum of sizes constant, i.e $\sum_j \lambda_j = 0.55$. We present the results in Figures 8 and 9.

We document a number of novel results. First, PI is mildly hump-shaped and generally decreases with size of the passive sector. Second, as the active oligopolist's size shrinks, her informa-

do not produce information, and that they care about price impact when trading. If these two characteristics are preserved, other formulations of passive investors (not being able to learn from prices, buying market shares, etc) will preserve our results.

tion pass-through goes up (up to a certain size), which improves price informativeness–consistent with intuition developed in previous sections. This effect is illustrated by the fixed-information-pass-through line in Figure 8, which does not exhibit the hump shape. Third, in terms of the learning channel, the active oligopolist is getting smaller and hence more specialized in learning. This reduces the number of assets she learns about, increases price informativeness of the high-supply assets, and reduces price informativeness of the low-supply assets. This effect is illustrated in Figure 9. Notably, without this reallocation of learning, the PI curve would have been higher and much more hump-shaped: the fixed learning channel curve in Figure 8 illustrates this point.

The cross-sectional patterns of price informativeness in Figure 9 bear additional notice in this experiment. In particular, the pattern of price informativeness emerging as the size of the passive sector goes up is heterogeneous across levels of asset supply. Price informativeness of large supply assets (high indexes, e.g., $6 - 10$) goes up as the active oligopolist specializes in them, while price informativeness of smaller supply assets decreases (low indexes, e.g., $1 - 5$). This result is reminiscent of a similar heterogeneity in the data documented in Farboodi, Matray, Veldkamp, and Venkateswaran (2020). Our framework links these responses to the rise of passive investing, which can be one of the channels driving the data trends.

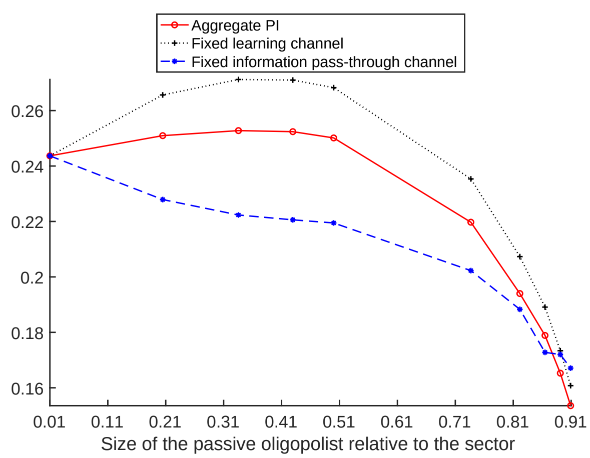

Figure 8: Price informativeness decomposition and the size of passive oligopolist.

5.4 The Role of Endogenous Learning

One of the novelties of our framework is that it features endogenous information choices together with quantity choices. The contrasting model with a fixed information structure is similar in

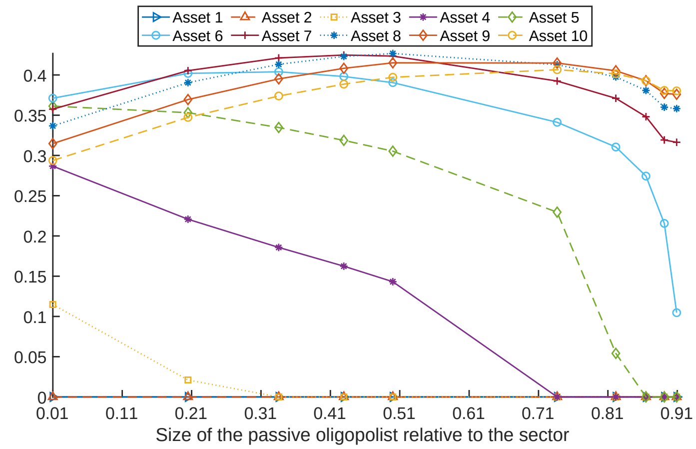

Figure 9: Asset-level price informativeness and the size of passive oligopolist.

spirit to Kyle (1989), in that the effect of market power on price informativeness depends entirely on the adjustment of quantities. In this section, we present numerical results that provide a comparison between the models. Figures 10–12 present the results from our three experiments for the benchmark and exogenous information models. In the exogenous information case, we endow oligopolists with the α_{ji} choices that are solutions for one of the benchmark model cases and eliminate the possibility of re-optimization along the learning dimension.

Figure 10 presents the comparison for the growth in overall sector size experiment. Depending on the choice of α_{ji} (case 1 or 2), one solution of the counterfactual model coincides with the benchmark model. However, comparison of the predictions of the three models implies that the specific choice of the information structure significantly impacts the size of the oligopoly sector that maximizes price informativeness. In particular, for case 1 the maximum is at 36% share of the market, and for case 2 it is at 45%. Compared to 61% for the benchmark model, fixing the information structure can lead to a understatement of the optimal oligopoly share by up to 40% relative to the endogenous information model.

The comparison for the concentration experiment is presented in Figure 11. In this case, the conclusions are even starker. Exogenous learning choices can produce an interior optimal concentration (case 1 in Figure 11 gives a maximum at 72%) while the benchmark model indicates that concentration always hurts price informativeness.

Finally, the results for the experiment of growth of the passive investor present similar conclusions to the size experiment (Figure 12). The two exogenous cases give a maximum of price infor-

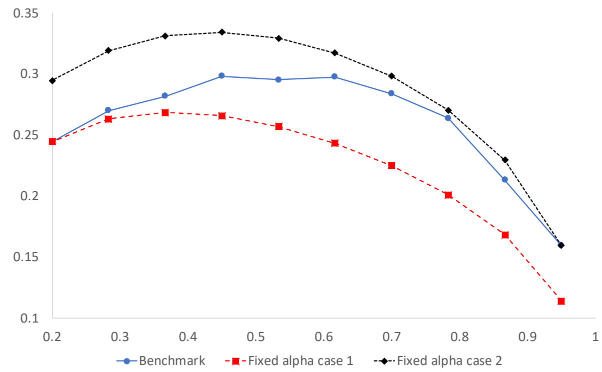

Figure 10: Price informativeness with endogenous (benchmark) and exogenous (fixed alpha) information choice and the oligopoly sector share of the market.

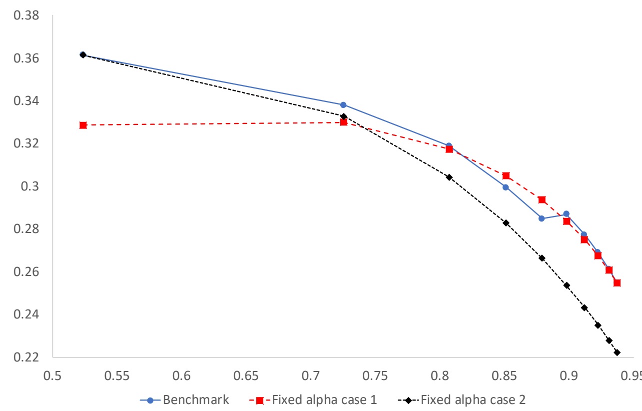

Figure 11: Price informativeness with endogenous (benchmark) and exogenous (fixed alpha) information choice and the size of the largest oligopolist's holdings.

mativeness at 50% and 42% of ownership share of the passive oligopolist relative to the oligopolistic sector. That amounts to an overstatement of the optimal passive oligopolist share by up to 50% relative to the endogenous information model-implied 33% share.

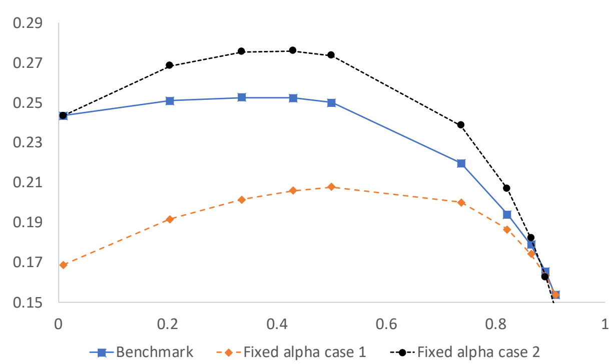

Figure 12: Price informativeness with endogenous (benchmark) and exogenous (fixed alpha) information choice and the size of passive oligopolist relative to the oligopolistic sector.

The main message from this experiment is that depending on the specific choice of the exogenous information structure, the conclusions about the impact of size, concentration, or passive ownership on price informativeness can be dramatically different. Hence, considering a fully endogenous adjustment of quantities *and* information choices when studying price informativeness is crucial for drawing conclusions about the efficiency of different ownership structures.

6 Concluding Remarks

Equities are overwhelmingly held by institutional investors, and ownership is especially concentrated among the largest investors. This skewed ownership structure has triggered an active discussion among financial regulators and industry participants over its implications for welfare and financial stability. Proponents of regulation have argued in favor of reduced power for large institutions, while critics of such reforms argue that the information such institutions imbue into prices makes market concentration a worthwhile tradeoff. In the absence of a well-specified economic model and a properly specified objective function, it is difficult to shed light on the argument and understand how to quantify the tradeoffs of a more concentrated marketplace, where concentration can occur in assets under management, as well as in capacity for fundamental research.

This paper takes a step towards addressing this issue by developing a general equilibrium model in which asymmetric information, asymmetric market power, and asset heterogeneity are important determinants of the informational efficiency that regulators might want to maximize. While regulators' objective functions can take different forms, we believe that the setting in which informational content of prices is of a planner's interest is appropriate to characterize the world of equity ownership. Further, our theory makes a methodological contribution in generalizing models of asymmetric information (building on, for example, Kyle (1989)), by explicitly modeling endogenously acquired information in the presence of asymmetric market power and nontrivial heterogeneities across investors and assets.

Our theoretical results suggest intermediate size of institutional sector maximizes information contained in prices. Our numerical simulations confirm that for ownership levels equal to those currently found in the U.S. (roughly 60%), average price efficiency is positively related to the levels of large ownership but negatively related to its concentration. Further, we show that average price informativeness across assets can be maximized for admissible values of ownership and concentration. This result suggests that policy makers should consider concentration in addition to size when interested in maximizing price efficiency.

Our model can be applied to settings with rich cross-section of assets, informational asymmetries across oligopolistic agents, and differences in market power. However, the model also comes with some important limitations. From a technical perspective, we largely focus on the strategic behavior of, and equilibrium effects triggered by, investors with market power. For expositional reasons, the model leaves out the more complicated considerations of the responses of atomistic investors, such as their learning from prices. Such considerations present important new challenges for theoretical researchers in the field. At the broad policy level, the model can be fruitfully used in a discussion of market transparency and access to information. At the same time, the model naturally abstracts from other important dimensions relevant for policy makers, such as investment costs or sectoral fund flows, as the size distribution is an input in our analysis. We also abstract from endogenous changes in market structure due to entry and exit, which could change the aggregate amount of information in the economy. We leave these issues for future research.

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A Appendix

A.1 Derivation of Equations (14) and (15)

The information choice solves

$$\max_{\{\widehat{\sigma}_{hi}^2\}_{i=1}^n} U_{0h} \equiv \frac{1}{2\rho}\sum_{i=1}^n \frac{E_{0h}\left(\widehat{\mu}_{hi} - rp_i\right)^2}{\widehat{\sigma}_{hi}^2} \tag{19}$$

subject to the relative entropy constraint

$$\prod_{i=1}^n \frac{\sigma_i^2}{\widehat{\sigma}_{hi}^2} \leq e^{2K_h}. \tag{20}$$

The objective can be written as

$$U_{0h} = \sum_{i=1}^n G_i \frac{\sigma_i^2}{\widehat{\sigma}_{hi}^2}, \tag{21}$$

where G_i is the gain of spending information capacity on asset i. We obtain a corner solution: each investor h learns about one asset among ones that maximize G_i. The gain to the competitive investors from learning about asset i is:

$$G_i \equiv \frac{\left(\overline{z} - ra_i\right)^2}{\sigma_i^2} + \left(1 - rb_i\right)^2 + r^2 c_i^2 \frac{\sigma_{xi}^2}{\sigma_i^2} - \frac{\widehat{\sigma}_{hi}^2}{\sigma_i^2}(1 - 2rb_i)$$

Hence, the gain from learning about a particular asset is the same across all competitive investors. To derive the above, note that the objective is:
$U_{0h} = \frac{1}{2\rho}\sum_{i=1}^n \frac{E_{0h}(\widehat{\mu}_{hi} - rp_i)^2}{\widehat{\sigma}_{hi}^2} = \frac{1}{2\rho}\sum_{i=1}^n \frac{\widehat{R}_i^2 + \widehat{V}_{hi}}{\widehat{\sigma}_{hi}^2}$, where

$\widehat{R}_i \equiv E_{0h}\left(\widehat{\mu}_{hi} - rp_i\right) = \overline{z} - r\overline{p}_i = \overline{z} - ra_i,$
$\widehat{V}_{hi} \equiv V_{0h}\left(\widehat{\mu}_{hi} - rp_i\right) = Var\left(\widehat{\mu}_{hi}\right) + r^2\sigma_{pi}^2 - 2rCov\left(\widehat{\mu}_{hi}, p_i\right).$

$Var\left(\widehat{\mu}_{hi}\right) = \sigma_i^2 - \widehat{\sigma}_{hi}^2.$
$\sigma_{pi}^2 = b_i^2\sigma_i^2 + c_i^2\sigma_{xi}^2.$

Posterior beliefs and prices are conditionally independent given payoffs, which implies:

$$\begin{aligned} Cov\left(\widehat{\mu}_{hi}, p_i\right) &= \frac{1}{\sigma_i^2} Cov\left(\widehat{\mu}_{hi}, z_i\right) Cov\left(z_i, p_i\right) \\ &= \frac{1}{\sigma_i^2}\left(\sigma_i^2 - \widehat{\sigma}_{hi}^2\right) Cov(\varepsilon_i, b_i\varepsilon_i) = \frac{1}{\sigma_i^2}\left(\sigma_i^2 - \widehat{\sigma}_{hi}^2\right) b_i\sigma_i^2 = \left(\sigma_i^2 - \widehat{\sigma}_{hi}^2\right) b_i \end{aligned}$$

Hence

$$\widehat{V}_{hi} = \sigma_i^2 - \widehat{\sigma}_{hi}^2 + r^2(b_i^2\sigma_i^2 + c_i^2\sigma_{xi}^2) - 2r\left(\sigma_i^2 - \widehat{\sigma}_{hi}^2\right) b_i.$$

Expected utility becomes $U_{0h} = \frac{1}{2\rho}\sum_{i=1}^n G_i \frac{\sigma_i^2}{\widehat{\sigma}_{hi}^2} - \frac{1}{2\rho}\sum_{i=1}^n (1 - 2rb_i)$. Ignoring constants, we get (14).

A.2 Derivation of Equation (6)

Market clearing for each asset i is

$$x_i = \sum_{j=1}^l \lambda_j q_{ji} + \lambda_0 q_{hi} = \sum_{j=1}^l \lambda_j q_{ji} + \lambda_0 \frac{\bar{z}_i - rp_i}{\rho\sigma_i^2} = \sum_{j=1}^l \lambda_j q_{ji} + \frac{\lambda_0}{\rho\sigma_i^2}\left[\bar{z}_i - rp_i\right]$$

Plugging (3) market clearing becomes, from the perspective of oligopolist k:

$$x_i = \lambda_k q_{ki} + \sum_{j=-k} \lambda_j(\beta_{0ji} + \beta_{1ji}s_{ji} - \beta_{2ji}rp_i) - \frac{\lambda_0}{\rho\sigma_i^2}rp_i + \frac{\lambda_0}{\rho\sigma_i^2}\bar{z}_i$$

collecting terms with the price gives

$$x_i = \lambda_k q_{ki} + \sum_{j=-k} \lambda_j(\beta_{0ji} + \beta_{1ji}s_{ji}) - p_i\left\{\frac{\lambda_0 r}{\rho\sigma_i^2} + \sum_{j=-k}\lambda_j r\beta_{2ji}\right\} + \frac{\lambda_0}{\rho\sigma_i^2}\bar{z}_i$$

$$p_i\left\{\frac{\lambda_0 r}{\rho\sigma_i^2} + \sum_{j=-k}\lambda_j r\beta_{2ji}\right\} = -x_i + \lambda_k q_{ki} + \sum_{j=-k}\lambda_j(\beta_{0ji} + \beta_{1ji}s_{ji}) + \frac{\lambda_0}{\rho\sigma_i^2}\bar{z}_i$$

which gives (6):

$$\frac{dp_i}{dq_{ki}} = \frac{\lambda_k}{\frac{\lambda_0 r}{\rho\sigma_i^2} + \sum_{j=-k}\lambda_j r\beta_{2ji}} = \frac{\rho\sigma_i^2\lambda_k}{\lambda_0 r + \rho\sigma_i^2\sum_{j=-k}\lambda_j r\beta_{2ji}}. \tag{22}$$

A.3 Derivation of Equations (7)-(9)

The price observed by oligopolist k is

$$p_i\left\{\frac{\lambda_0 r}{\rho\sigma_i^2} + \sum_{j=1}\lambda_j r\beta_{2ji}\right\} = -x_i + \lambda_k\beta_{1ki}s_{ki} + \sum_{j=1}^{l}\lambda_j\beta_{0ji} + \sum_{j=-k}\lambda_j\beta_{1ji}s_{ji} + \frac{\lambda_0}{\rho\sigma_i^2}\bar{z}_i.$$

Dividing by $\lambda_0/\rho\sigma_i^2$, we obtain

$$rp_i\left\{1 + \frac{\rho\sigma_i^2}{\lambda_0}\sum_{j=-k}\lambda_j\beta_{2ji}\right\} = -x_i\frac{\rho\sigma_i^2}{\lambda_0} + \frac{\rho\sigma_i^2}{\lambda_0}[\lambda_k\beta_{1ki}s_{ki} + \sum_{j=1}^{l}\lambda_j\beta_{0ji}] + \frac{\rho\sigma_i^2}{\lambda_0}\sum_{j=-k}\lambda_j\beta_{1ji}s_{ji} + \bar{z}$$

With $\Delta_i \equiv 1 + \frac{\rho\sigma_i^2}{\lambda_0}\sum_{j=1}^{l}\lambda_j\beta_{2ji}$, we can write:

$$rp_i = -\frac{1}{\Delta_i}x_i\frac{\rho\sigma_i^2}{\lambda_0} + \frac{1}{\Delta_i}\frac{\rho\sigma_i^2}{\lambda_0}\lambda_k\beta_{1ki}s_{ki} + \frac{1}{\Delta_i}\frac{\rho\sigma_i^2}{\lambda_0}\sum_{j=1}^{l}\lambda_j\beta_{0ji} + \frac{1}{\Delta_i}\frac{\rho\sigma_i^2}{\lambda_0}\sum_{j=-k}\lambda_j\beta_{1ji}s_{ji} + \frac{1}{\Delta_i}\bar{z}$$

Then:

$$rcov_k(z_i, p_i) = \frac{1}{\Delta_i}\frac{\rho\sigma_i^2}{\lambda_0}\sum_{j=-k}\lambda_j\beta_{1ji}cov_k(s_{ji}, z_i) = \frac{1}{\Delta_i}\frac{\rho\sigma_i^2}{\lambda_0}\sum_{j=-k}\lambda_j\beta_{1ji}cov_k((1 - \frac{1}{\alpha_{ji}})\varepsilon_i + \zeta_{ji}, \varepsilon_i) =$$

$$= \frac{1}{\Delta_i}\frac{\rho\sigma_i^2}{\lambda_0}\sum_{j=-k}\lambda_j\beta_{1ji}cov_k((1 - \frac{1}{\alpha_{ji}})\varepsilon_i, \varepsilon_i) = \frac{1}{\Delta_i}\frac{\rho\sigma_i^2}{\lambda_0}\sum_{j=-k}\lambda_j\beta_{1ji}(1 - 1/\alpha_{ji})\eta_{ki}^2$$

$$= \frac{1}{\Delta_i}\frac{\rho\sigma_i^2}{\lambda_0}\frac{1}{\alpha_{ki}}\sum_{j=-k}\lambda_j\beta_{1ji}(1 - 1/\alpha_{ji})\sigma_i^2$$

Notably, the last variance term is associated with the beliefs of oligopolist k, not j.

$$r^2 var_k(p_i) = (\frac{1}{\Delta_i}\frac{\rho\sigma_i^2}{\lambda_0})^2\sigma_{xi}^2 + (\frac{1}{\Delta_i}\frac{\rho\sigma_i^2}{\lambda_0})^2 var_k(\sum_{j=-k}\lambda_j\beta_{1ji}(\bar{z}_i + (1 - \frac{1}{\alpha_{ji}})\varepsilon_i + \zeta_{ji})) =$$

$$= (\frac{1}{\Delta_i}\frac{\rho\sigma_i^2}{\lambda_0})^2\sigma_{xi}^2 + (\frac{1}{\Delta_i}\frac{\rho\sigma_i^2}{\lambda_0})^2 var_k(\varepsilon_i \sum_{j=-k} \lambda_j\beta_{1ji}(1-\frac{1}{\alpha_{ji}}) + (\frac{1}{\Delta_i}\frac{\rho\sigma_i^2}{\lambda_0})^2 \sum_{j=-k} \lambda_j\beta_{1ji}\zeta_{ji})) =$$

$$= (\frac{1}{\Delta_i}\frac{\rho\sigma_i^2}{\lambda_0})^2\sigma_{xi}^2 + (\frac{1}{\Delta_i}\frac{\rho\sigma_i^2}{\lambda_0})^2 \frac{1}{\alpha_{ki}}\sigma_i^2 \left[\sum_{j=-k} \lambda_j\beta_{1ji}(1-\frac{1}{\alpha_{ji}})\right]^2 + (\frac{1}{\Delta_i}\frac{\rho\sigma_i^2}{\lambda_0})^2 \sum_{j=-k} \lambda_j^2\beta_{1ji}^2(1-\frac{1}{\alpha_{ji}})\frac{1}{\alpha_{ji}}\sigma_i^2) =$$

$E_{ki}[p_i|s_{ki}]$ is given by (omitting the conditioning on the signal notation)

$$rE_{ki}[p_i] = -\frac{1}{\Delta_i}\bar{x}_i\frac{\rho\sigma_i^2}{\lambda_0} + \frac{1}{\Delta_i}\frac{\rho\sigma_i^2}{\lambda_0}\lambda_k\beta_{1ki}s_{ki} + \frac{1}{\Delta_i}\frac{\rho\sigma_i^2}{\lambda_0}\sum_{j=1}^{l}\lambda_j\beta_{0ji} + \frac{1}{\Delta_i}\frac{\rho\sigma_i^2}{\lambda_0}\sum_{j=-k}\lambda_j\beta_{1ji}(\bar{z}_i + (1-1/\alpha_{ji})(s_{ki}-\bar{z}_i)) + \frac{1}{\Delta_i}\bar{z}$$

$$rE_{ki}[p_i] = s_{ki}\frac{1}{\Delta_i}\frac{\rho\sigma_i^2}{\lambda_0}\left[\lambda_k\beta_{1ki} + \sum_{j\neq k}\lambda_j\beta_{1ji}(1-1/\alpha_{ji})\right] - \frac{1}{\Delta_i}\bar{x}_i\frac{\rho\sigma_i^2}{\lambda_0} + \frac{1}{\Delta_i}\frac{\rho\sigma_i^2}{\lambda_0}\sum_{j=1}^{l}\lambda_j\beta_{0ji} + \frac{1}{\Delta_i}\frac{\rho\sigma_i^2}{\lambda_0}\sum_{j=-k}\lambda_j\beta_{1ji}1/\alpha_{ji}\bar{z}_i + \frac{1}{\Delta_i}\bar{z}$$

Denote:

$$\Gamma_{ki} = -\frac{1}{\Delta_i}\bar{x}_i\frac{\rho\sigma_i^2}{\lambda_0} + \frac{1}{\Delta_i}\frac{\rho\sigma_i^2}{\lambda_0}\sum_{j=1}^{l}\lambda_j\beta_{0ji} + \frac{1}{\Delta_i}\frac{\rho\sigma_i^2}{\lambda_0}\sum_{j=-k}\lambda_j\beta_{1ji}1/\alpha_{ji}\bar{z}_i + \frac{1}{\Delta_i}\bar{z}$$

and

$$\theta_{ki} = \frac{1}{\Delta_i}\frac{\rho\sigma_i^2}{\lambda_0}\left[\lambda_k\beta_{1ki} + \sum_{j\neq k}\lambda_j\beta_{1ji}(1-1/\alpha_{ji})\right]$$

Using these results in (5), we get

$$q_{ji} = \frac{\hat{\mu}_{ji} - rp_i}{\rho\hat{\sigma}_{ji}^2 + r\frac{dp_i}{dq_{ji}}},$$

$$\mu_{ji} = s_{ji} + \frac{cov_j(z_i, p_i)}{\sigma_{pji}^2}(p_i - E_j[p_i])$$

$$\hat{\sigma}_{ji}^2 = \frac{1}{\alpha_{ji}}\sigma_i^2 - \frac{cov_j^2(z_i, p_i)}{\sigma_{pji}^2}$$

Now, note that the covariance and variance of price, though oligopolist-specific, do not depend on s_{ji} or p_i, p_i is the observation, and $E_j[p_i]$ depends on the signal s_{ji}. Denoting $cov_j(p_i)/\sigma_{pji}^2$ as γ_{ji}, we can write

$$q_{ji}(\rho\hat{\sigma}_{ji}^2 + r\frac{dp_i}{dq_{ji}}) = s_{ji} + \gamma_{ji}p_i - rp_i - \gamma_{ji}(s_{ji}\theta_{ji} + \Gamma_{ji})$$

$$q_{ji}(\rho\hat{\sigma}_{ji}^2 + r\frac{dp_i}{dq_{ji}}) = -\gamma_{ji}\Gamma_{ji} + s_{ji}(1-\gamma_{ji}\theta_{ji}) - r(1-\gamma_{ji}/r)p_i$$

Given that, we obtain the fixed point for betas:

$$\begin{aligned}
\beta_{0ji} &= -\gamma_{ji}\Gamma_{ji} \\
\beta_{1ji} &= \frac{(1-\gamma_{ji}\theta_{ji})}{\rho\hat{\sigma}_{ji}^2 + r\frac{dp_i}{dq_{ji}}}, \\
\beta_{2ji} &= \frac{1-\gamma_{ji}/r}{\rho\hat{\sigma}_{ji}^2 + r\frac{dp_i}{dq_{ji}}}, \\
\frac{dp_i}{dq_{ki}} &= = \frac{\rho\sigma_i^2\lambda_k}{\lambda_0 r(1+\Phi_i) + \rho\sigma_i^2\sum_{j=-k}\lambda_j r\beta_{2ji}}.
\end{aligned}$$

A.4 Utility Maximization

The ex-ante information decision maximizes:

$$E_0 U_j = \sum_{i=1}^{n} E_0(\hat{\mu}_{ji} - rp_i)^2 \frac{\frac{\rho}{2}\hat{\sigma}_{ji}^2 + r\frac{dp_i}{dq_{ji}}}{(\rho\hat{\sigma}_{ji}^2 + r\frac{dp_i}{dq_{ji}})^2}$$

We will use

$$rp_i = -\frac{1}{\Delta_i}x_i\frac{\rho\sigma_i^2}{\lambda_0} + \frac{1}{\Delta_i}\frac{\rho\sigma_i^2}{\lambda_0}\sum_{j=1}^{l}\lambda_j\beta_{0ji} + \frac{1}{\Delta_i}\frac{\rho\sigma_i^2}{\lambda_0}\sum_{j=1}^{l}\lambda_j\beta_{1ji}s_{ji} + \frac{1}{\Delta_i}\bar{z}$$

where

$$\Delta_i = \left(1 + \frac{\rho\sigma_i^2}{\lambda_0}\sum_{j=1}^{l}\lambda_j\beta_{2ji}\right)$$

We compute $E_{0j}(\widehat{\mu}_{ji} - rp_i)^2 = \widehat{R}_i^2 + \widehat{V}_{ji}$, where $\widehat{R}_i$ and $\widehat{V}_{ji}$ denote the ex-ante mean and variance of expected excess returns,

$$\hat{R}_i = E_{ji0}(\hat{\mu}_{ji} - rp_i) = \bar{z}_i + \frac{1}{\Delta_i}\bar{x}_i\frac{\rho\sigma_i^2}{\lambda_0} - \frac{1}{\Delta_i}\frac{\rho\sigma_i^2}{\lambda_0}\sum_{j=1}^{l}\lambda_j\beta_{0ji} - \frac{1}{\Delta_i}\frac{\rho\sigma_i^2}{\lambda_0}\sum_{j=1}^{l}\lambda_j\beta_{1ji}\bar{z}_i - \frac{1}{\Delta_i}\bar{z}_i =$$

$$= \frac{1}{\Delta_i}\bar{x}_i\frac{\rho\sigma_i^2}{\lambda_0} - \frac{1}{\Delta_i}\frac{\rho\sigma_i^2}{\lambda_0}\sum_{j=1}^{l}\lambda_j\beta_{0ji} + \bar{z}_i(1 - \frac{1 + \frac{\rho\sigma_i^2}{\lambda_0}\sum_{j=1}^{l}\lambda_j\beta_{1ji}}{\Delta_i})$$

We now compute

$$\hat{V}_{ji} = var_0(\mu - rp_i)$$

$$var_0(\hat{\mu}_{ji} - rp_i) = var_0(\hat{\mu}_{ji}) + var_0(rp_i) - 2rcov(\hat{\mu}_{ji}, p_i)$$

We have

$$\begin{aligned}
var_0(\hat{\mu}_{ji}) &= var_0(s_{ji}+\gamma_{ji}(p_i - E_j[p_i])) = var_0(s_{ji}) + \gamma_{ji}^2 var(p_i) - 2\gamma_{ji} cov(s_{ji}, p_i) = \\
&= (1-1/\alpha_{ji})\sigma_i^2 + \frac{\gamma_{ji}^2}{r^2}\left(\frac{1}{\Delta_i}\frac{\rho\sigma_i^2}{\lambda_0}\right)^2 \left(\sigma_{ix}^2 + \sum_{k=1}^{l}\lambda_k^2\beta_{1ki}^2(1-1/\alpha_{ki})\sigma_i^2\right) \\
&- 2\gamma_{ji}\frac{\rho\sigma_i^2}{r\lambda_0\Delta_i}\lambda_j\beta_{1ji}(1-1/\alpha_{ji})\sigma_i^2 \\
var_0(rp_i) &= \left(\frac{1}{\Delta_i}\frac{\rho\sigma_i^2}{\lambda_0}\right)^2 \left(\sigma_{ix}^2 + \sum_{k=1}^{l}\lambda_k^2\beta_{1ki}^2(1-1/\alpha_{ki})\sigma_i^2\right) \\
2rcov(\mu_{ji}, p_i) &= 2rcov(s_{ji}+\gamma_{ji}(p_i - E_j[p_i]), p_i) = 2rcov(s_{ji}, p_i) + 2r\gamma_{ji} var(p_i) = \\
&= 2\frac{\rho\sigma_i^2}{\lambda_0\Delta_i}\lambda_j\beta_{1ji}(1-1/\alpha_{ji})\sigma_i^2 + 2\frac{\gamma_{ji}}{r}\left(\frac{1}{\Delta_i}\frac{\rho\sigma_i^2}{\lambda_0}\right)^2 \left(\sigma_{ix}^2 + \sum_{k=1}^{l}\lambda_k^2\beta_{1ki}^2(1-1/\alpha_{ki})\sigma_i^2\right)
\end{aligned}$$

Summing up:

$$\begin{aligned}
\hat{V}_{ji} &= (1-1/\alpha_{ji})\sigma_i^2 + \left(\frac{\gamma_{ji}^2}{r^2} + 1 - 2\frac{\gamma_{ji}}{r}\right)\left(\frac{1}{\Delta_i}\frac{\rho\sigma_i^2}{\lambda_0}\right)^2 \left(\sigma_{ix}^2 + \sum_{k=1}^{l}\lambda_k^2\beta_{1ki}^2(1-1/\alpha_{ki})\sigma_i^2\right) \\
&- 2(\frac{\gamma_{ji}}{r}+1)\frac{\rho\sigma_i^2}{\lambda_0\Delta_i}\lambda_j\beta_{1ji}(1-1/\alpha_{ji})\sigma_i^2 \qquad (23) \\
\hat{V}_{ji} &= (1-1/\alpha_{ji})\sigma_i^2 + \left(\frac{\gamma_{ji}^2}{r^2} + 1 - 2\frac{\gamma_{ji}}{r}\right)\left(\frac{1}{\Delta_i}\frac{\rho\sigma_i^2}{\lambda_0}\right)^2 \left(\sigma_{ix}^2 + \sum_{k=1}^{l}\lambda_k^2\beta_{1ki}^2(1-1/\alpha_{ki})\sigma_i^2\right) \\
&- 2(\frac{\gamma_{ji}}{r}+1)\frac{\rho\sigma_i^2}{\lambda_0\Delta_i}\lambda_j\beta_{1ji}(1-1/\alpha_{ji})\sigma_i^2 \qquad (24)
\end{aligned}$$

A.5 Derivation of the Monopolist's Utility (17)

Under the monopoly structure and no learning from prices, we have: $\gamma_{ji} = 0$ and also $\beta_1 = \beta_2$ and $dp/dq = \rho\sigma_i^2\lambda_1/(r\lambda_0)$, such that

$$\frac{r}{\rho}\frac{dp}{dq} = \sigma_i^2\frac{\lambda_1}{\lambda_0} \equiv \sigma_i^2\hat{\lambda}$$

The ex-ante information decision maximizes:

$$E_0 U_j = \frac{1}{2\rho}\sum_{i=1}^{n} E_0(\hat{\mu}_{ji} - rp_i)^2 \frac{\hat{\sigma}_{ji}^2 + 2r\frac{dp_i}{\rho dq_{ji}}}{(\hat{\sigma}_{ji}^2 + r\frac{dp_i}{\rho dq_{ji}})^2}$$

which becomes

$$E_0 U_j = \frac{1}{2\rho}\sum_{i=1}^{n} E_0(\hat{\mu}_{ji} - rp_i)^2 \frac{\hat{\sigma}_{ji}^2 + 2\sigma_i^2\hat{\lambda}}{(\hat{\sigma}_{ji}^2 + \sigma_i^2\hat{\lambda})^2}$$

We will use

$$rp_i = -\frac{1}{\Delta_i}x_i\frac{\rho\sigma_i^2}{\lambda_0} + \frac{1}{\Delta_i}\frac{\rho\sigma_i^2}{\lambda_0}\sum_{j=1}^{l}\lambda_j\beta_{0ji} + \frac{1}{\Delta_i}\frac{\rho\sigma_i^2}{\lambda_0}\sum_{j=1}^{l}\lambda_j\beta_{1ji}s_{ji} + \frac{1}{\Delta_i}\bar{z}$$

where

$$\Delta_i = \left(1 + \frac{\rho\sigma_i^2}{\lambda_0}\sum_{j=1}^{l}\lambda_j\beta_{2ji}\right)$$

As before, we compute $E_{0j}(\widehat{\mu}_{ji} - rp_i)^2 = \widehat{R}_i^2 + \widehat{V}_{ji}$, where $\widehat{R}_i$ and $\widehat{V}_{ji}$ denote the ex-ante mean and variance of expected excess returns,

$$\hat{R}_i = E_{ji0}(\hat{\mu}_{ji} - rp_i) = \bar{z}_i + \frac{1}{\Delta_i}\bar{x}_i\frac{\rho\sigma_i^2}{\lambda_0} - \frac{1}{\Delta_i}\frac{\rho\sigma_i^2}{\lambda_0}\sum_{j=1}^{l}\lambda_j\beta_{0ji} - \frac{1}{\Delta_i}\frac{\rho\sigma_i^2}{\lambda_0}\sum_{j=1}^{l}\lambda_j\beta_{1ji}\bar{z}_i - \frac{1}{\Delta_i}\bar{z}_i =$$

$$= \frac{1}{\Delta_i}\bar{x}_i\frac{\rho\sigma_i^2}{\lambda_0} - \frac{1}{\Delta_i}\frac{\rho\sigma_i^2}{\lambda_0}\sum_{j=1}^{l}\lambda_j\beta_{0ji} + \bar{z}_i(1 - \frac{1 + \frac{\rho\sigma_i^2}{\lambda_0}\sum_{j=1}^{l}\lambda_j\beta_{1ji}}{\Delta_i})$$

$$= \frac{1}{\Delta_i}\bar{x}_i\frac{\rho\sigma_i^2}{\lambda_0}$$

We now compute

$$\hat{V}_{ji} = var_0(\mu - rp_i)$$

$$var_0(\hat{\mu}_{ji} - rp_i) = var_0(\hat{\mu}_{ji}) + var_0(rp_i) - 2rcov(\mu_{ji}, p_i)$$

We have

$$\begin{aligned}
var_0(\mu_{ji}) &= var_0(s_{ji} + \gamma_{ji}(p_i - E_j[p_i]) = var_0(s_{ji}) + \gamma_{ji}^2 var(p_i) - 2\gamma_{ji}cov(s_{ji}, p_i) = \\
&= (1 - 1/\alpha_{ji})\sigma_i^2 \\
var_0(rp_i) &= \left(\frac{1}{\Delta_i}\frac{\rho\sigma_i^2}{\lambda_0}\right)^2\left(\sigma_{ix}^2 + \lambda_1^2\beta_i^2(1 - 1/\alpha_{ji})\sigma_i^2\right) \\
2rcov(\hat{\mu}_{ji}, p_i) &= 2rcov(s_{ji} + \gamma_{ji}(p_i - E_j[p_i]), p_i) = 2rcov(s_{ji}, p_i) = \\
&= 2\frac{\rho\sigma_i^2}{\lambda_0\Delta_i}\lambda_j\beta_i(1 - 1/\alpha_{ji})\sigma_i^2
\end{aligned}$$

Summing up:

$$\begin{aligned}
\hat{V}_i &= (1 - 1/\alpha_{ji})\sigma_i^2(1 + \left(\frac{1}{\Delta_i}\frac{\rho\sigma_i^2}{\lambda_0}\right)^2(\frac{\sigma_{xi}^2}{(1 - 1/\alpha_i)\sigma_i^2} + \lambda_1^2\beta_i^2) - 2\frac{1}{\Delta_i}\frac{\rho\sigma_i^2}{\lambda_0}\lambda_1\beta_i) \\
&= (1 - 1/\alpha_{ji})\sigma_i^2[(1 - \frac{1}{\Delta_i}\frac{\rho\sigma_i^2}{\lambda_0}\lambda_1\beta_i)^2 + (\frac{1}{\Delta_i}\frac{\rho\sigma_i^2}{\lambda_0})^2\frac{\sigma_{xi}^2}{(1 - 1/\alpha_i)\sigma_i^2}] \qquad (25) \\
&= (1 - 1/\alpha_{ji})\sigma_i^2[\frac{1}{\Delta_i^2} + (\frac{1}{\Delta_i}\frac{\rho\sigma_i^2}{\lambda_0})^2\frac{\sigma_{xi}^2}{(1 - 1/\alpha_i)\sigma_i^2}] \qquad (26) \\
& \qquad (27)
\end{aligned}$$

So, $R_i^2 + V_i$ is:

$$(\frac{1}{\Delta_i}\bar{x}_i\frac{\rho\sigma_i^2}{\lambda_0})^2 + (\sigma_i^2 - \hat{\sigma}_i^2)[\frac{1}{\Delta_i^2} + \frac{(\frac{1}{\Delta_i}\frac{\rho\sigma_i^2}{\lambda_0})^2\sigma_{xi}^2}{(\sigma_i^2 - \hat{\sigma}_i^2)}]$$

$$= \frac{1}{\Delta_i^2}[\frac{\rho^2\sigma_i^4}{\lambda_0^2}(\bar{x}_i^2 + \sigma_{xi}^2) + (\sigma_i^2 - \hat{\sigma}_i^2)]$$

Now,

$$\Delta_i = 1 + \frac{\rho\sigma_i^2}{\lambda_0}\beta_i\lambda_1$$

and

$$rdp/dq = \rho\sigma_i^2\lambda_1/(\lambda_0)$$

and

$$\beta = \frac{1}{\rho\hat{\sigma}_i^2 + rdp/dq}$$

so that

$$\Delta = \frac{\hat{\sigma}_i^2 + 2\sigma_i^2\lambda_1/\lambda_0}{\hat{\sigma}_i^2 + \sigma_i^2\lambda_1/\lambda_0}$$

so that utility is

$$E_0U_j = \frac{1}{2\rho}\sum_{i=1}^{n}\frac{(\hat{\sigma}_i^2 + \sigma_i^2\lambda_1/\lambda_0)^2}{(\hat{\sigma}_i^2 + 2\sigma_i^2\lambda_1/\lambda_0)^2}[\frac{\rho^2\sigma_i^4}{\lambda_0^2}(\bar{x}_i^2 + \sigma_{xi}^2) + (\sigma_i^2 - \hat{\sigma}_i^2)]\frac{\hat{\sigma}_{ji}^2 + 2\sigma_i^2\hat{\lambda}}{(\hat{\sigma}_{ji}^2 + \sigma_i^2\hat{\lambda})^2}$$

$$= \frac{1}{2\rho}\sum_{i=1}^{n}[\frac{\rho^2\sigma_i^4}{\lambda_0^2}(\bar{x}_i^2 + \sigma_{xi}^2) + (\sigma_i^2 - \hat{\sigma}_i^2)]\frac{1}{(\hat{\sigma}_i^2 + 2\sigma_i^2\hat{\lambda})}$$

which gives (17)

$$= \frac{1}{2\rho}\sum_{i=1}^{n}[\frac{\rho^2\sigma_i^2}{\lambda_0^2}(\bar{x}_i^2 + \sigma_{xi}^2)\alpha_i + \alpha_i - 1]\frac{1}{\lambda_0 + 2\lambda_1\alpha_i}$$

A.6 Derivation of Equation (10)

$$rp_i = -\frac{1}{\Delta_i}x_i\frac{\rho\sigma_i^2}{\lambda_0} + \frac{1}{\Delta_i}\frac{\rho\sigma_i^2}{\lambda_0}\sum_{j=1}^{l}\lambda_j\beta_{0ji} + \frac{1}{\Delta_i}\frac{\rho\sigma_i^2}{\lambda_0}\sum_{j=1}^{l}\lambda_j\beta_{1ji}(\bar{z}_i + (1 - 1/\alpha_{ji})\varepsilon_i - \zeta_{ji}) + \frac{1}{\Delta_i}\bar{z}$$

Given that, we have

$$cov(p_i, z_i) = \sigma_i^2\frac{1}{r\Delta_i}\frac{\rho\sigma_i^2}{\lambda_0}\sum_{j}^{l}\lambda_j\beta_{1ji}\frac{\alpha_{ji} - 1}{\alpha_{ji}}$$

and

$$var(p_i) = \sigma_i^2\frac{1}{(r\Delta_i)^2}(\frac{\rho\sigma_i^2}{\lambda_0})^2\left[\frac{\sigma_{xi}^2}{\sigma_i^2} + \left[\sum_{j}^{l}\lambda_j\beta_{1ji}(1 - \frac{1}{\alpha_{ji}})\right]^2 + \sum_{j}^{l}\lambda_j^2\beta_{1ji}^2\frac{\alpha_{ji} - 1}{\alpha_{ji}^2}\right]$$

Then, PI equals (the coefficient $\frac{1}{(r\Delta_i}\frac{\rho\sigma_i^2}{\lambda_0}$ cancels out)

$$PI_i = \frac{cov(p_i, z_i)}{\sigma_{pi}} = \frac{\sigma_i\sum_j^l\lambda_j\beta_{1ji}\frac{\alpha_{ji}-1}{\alpha_{ji}}}{\sqrt{\frac{\sigma_{xi}^2}{\sigma_i^2} + \left[\sum_j^l\lambda_j\beta_{1ji}(1 - \frac{1}{\alpha_{ji}})\right]^2 + \sum_j^l\lambda_j^2\beta_{1ji}^2\frac{\alpha_{ji}-1}{\alpha_{ji}^2}}}$$

Notice that $\lambda_j\beta_{1ji} = \frac{\partial\lambda_j q_{ji}}{\partial s_{ji}}$ is the reaction of the total quantity an oligopolist is purchasing with respect to the private signal, which we term the information pass-through. Defining information pass-through as

$$\omega_{ji} \equiv \frac{\partial\lambda_j q_{ji}}{\partial s_{ji}}.$$

results in equation (10)

$$PI_i = \frac{cov(p_i, z_i)}{\sigma_{pi}} = \frac{\sigma_i\sum_j^l\omega_{ji}\frac{\alpha_{ji}-1}{\alpha_{ji}}}{\sqrt{\frac{\sigma_{xi}^2}{\sigma_i^2} + \left[\sum_j^l\omega_{ji}(1 - \frac{1}{\alpha_{ji}})\right]^2 + \sum_j^l\omega_{ji}^2\frac{\alpha_{ji}-1}{\alpha_{ji}^2}}}.$$

A.7 Proof of Proposition 2

Proof. (i) $\frac{\partial M_i}{\partial \alpha_i} = \frac{(L_i+1-\lambda_1^2)(\lambda_0+2\lambda_1\alpha_i)}{(\lambda_0+2\lambda_1\alpha_i)^3} - \alpha_i \frac{(L_i+1-\lambda_1^2)4\lambda_1}{(\lambda_0+2\lambda_1\alpha_i)^3}$, which is equal in sign to

$$1 - \lambda_1 - 2\lambda_1\alpha_i.$$

Hence, given K, for λ_1 such that

$$\lambda_1 < \frac{1}{1+2e^{2K}} \equiv \underline{\lambda}$$

M_i is strictly increasing for all i. The M_is also always preserve the rank implied by L_i and therefore, the monopolist will learn only about the asset with the highest L_i.

(ii) By the analysis in (i), for $\lambda_1 > 1/3$, M_i is strictly decreasing in α_{1i} for all i. Moreover, for any two assets that are learned about, i and k, it has to be true that

$$\frac{L_i + 1 - \lambda_1^2}{(1-\lambda_1+2\lambda_1\alpha_i)^2}\alpha_i = \frac{L_k + 1 - \lambda_1^2}{(1-\lambda_1+2\lambda_1\alpha_k)^2}\alpha_k$$

Define the common value, which depends on λ_1 and K, as $\bar{M}(\lambda_1, K)$. Suppose that the monopolist learns about $N < n$ assets. We know that

$$\lim_{\lambda_1 \to 1} \bar{M}(\lambda_1, K) \equiv \bar{M} = \frac{L_i}{4\alpha_i} = \frac{L_k}{4\alpha_k} \tag{28}$$

and

$$N \log(\bar{M}) = \sum_{i=1}^{N} \log(L_i) - \sum_{i=1}^{N} \log(\alpha_i) = \sum_{i=1}^{N} \log L_i - 2K,$$

Suppose that asset j is never learned about. Then it must be true that for all λ_1, K, $\bar{M}(\lambda_1, K) > \frac{L_j+1-\lambda_1^2}{(1+\lambda_1)^2}$. That condition is violated for any asset with positive L_j and $K > \frac{1}{2}\sum_{i=1}^{n}\log(L_i)$. This defines $\bar{K} = \frac{1}{2}\sum_{i=1}^{n}\log(L_i)$. Additionally,

$$\frac{d\bar{M}(\lambda_1, K)}{d\lambda_1} = \frac{\partial \bar{M}(\lambda_1, K)}{\partial \lambda_1} + \frac{\partial \bar{M}(\lambda_1, K)}{\partial \alpha_i}\frac{d\alpha_i}{d\lambda_1} < 0. \tag{29}$$

To see this, note first that the two partial derivatives are negative. The term $d\alpha_i/d\lambda_1$ can be positive or negative, depending on the asset. However, even if it is negative, its effect cannot in equilibrium change the sign of (29). If it did, i.e. if $\bar{M}(\lambda_1, K)$ increased after an increase in λ_1, that would mean that M_i went up for all actively traded assets, and that can only be achieved if α_{1i} goes down for all actively traded assets, clearly violating optimality. This argument does not rely on the number of assets that are learned about being constant.

Equation (29) implies that assets are learned about according to ranking by L_i. As λ_1 increases the monopolist learns first about the highest-L_i asset, then the second highest L_i, etc. until all assets are learned about. A sufficient condition for that is $K > \bar{K}$ and positive L_is. Then all assets are learned about as λ_1 approaches 1, but is still bounded away from 1. The specific $\lambda_1(N)$ are defined by:

$$\frac{L_{N-1} + 1 - \lambda_1^2}{(1-\lambda_1+2\lambda_1\alpha_{1(N-1)})^2}\alpha_{1(N-1)} = \frac{L_N + 1 - \lambda_1^2}{(1+\lambda_1)^2}$$

which exists by monotonicity of $\bar{M}(\lambda_1, \bar{K})$ and (28).

□

A.8 Proof of Proposition 3

Proof. For any $\lambda_1 < \underline{\lambda}$, where $\underline{\lambda}$ is defined in Lemma 2, $\frac{d\alpha_{1i}}{d\lambda_1} = 0$ (it is e^{2K} for asset $i = 1$ and 1 otherwise. In such case,

$$\frac{d\lambda_1\beta_i}{d\lambda_1} = \frac{\partial\lambda_1\beta_i}{\partial\lambda_1} = \frac{1 - 2\lambda_1 - \lambda_1^2\alpha_i}{(1 - \lambda_1 + \lambda_1\alpha_i)^2}$$

Evaluating, we have that $\frac{d\lambda_1\beta_i}{d\lambda_1} > 0$ for $\lambda_1 < \frac{\sqrt{\alpha+1}-1}{\alpha}$ (which is strictly less than 1 and strictly more than 0). We have that information pass-through is strictly increasing for $\lambda_1 < \lambda_L \equiv \min\{\underline{\lambda}, \frac{\sqrt{e^{2K}+1}-1}{e^{2K}}\}$.
For large $\lambda_1 (> \sqrt{2} - 1)$, we have:

$$\frac{d\lambda_1\beta_i}{d\lambda_1} = \frac{\partial\lambda_1\beta_i}{\partial\lambda_1} + \frac{\partial\lambda_1\beta_i}{\partial\alpha_i}\frac{d\alpha_i}{d\lambda_1},$$

where

$$\frac{\partial\lambda_1\beta_i}{\partial\lambda_1} = \frac{1 - 2\lambda_1 - \lambda_1^2\alpha_i}{(1 - \lambda_1 + \lambda_1\alpha_i)^2} < 0,$$

$$\frac{\partial\lambda_1\beta_i}{\partial\alpha_i} = \frac{\lambda_1(1 - \lambda_1)^2}{(1 - \lambda_1 + \lambda_1\alpha_i)^2} \geq 0$$

Additionally,

$$\lim_{\lambda_1 \to 1} \frac{\partial\lambda_1\beta_i}{\partial\lambda_1} = -\frac{1 + \alpha_i}{\alpha_i^2} < -\frac{1 + e^{2K}}{e^{4K}},$$

and so it is bounded away from 0, and

$$\lim_{\lambda_1 \to 1} \frac{\partial\lambda_1\beta_i}{\partial\alpha_i} = 0.$$

By arguments underlying the proof of Lemma 2, $d\alpha_i/d\lambda_1$ is bounded from above and below. If it were not bounded from below, then it would have to be the case that $\frac{d\bar{M}(\lambda_1,K)}{d\lambda_1}$ in (29) be positive for some i and some value of λ_1, which creates a contradiction as it would mean that for that value of λ_1, the capacity constraint is slack, violating optimality. The immediate consequence is that it is also bounded from above as otherwise the capacity constraint $\sum_i \log(\alpha_i) \leq 2K$ would be violated for some λ_1. Based on these results, we can conclude that there exists a cutoff value $\lambda_H < 1$ such that for all $\lambda_1 > \lambda_H$, we have $\frac{d\lambda_1\beta_i}{d\lambda_1} < 0$. The non-negativity and zero values at points $\lambda_1 \in \{0, 1\}$ follow trivially from the equation. □

A.9 Proof of Corollary 2

Proof. We have

$$\frac{dPI}{d\lambda_1} = \frac{\partial PI}{\partial\alpha_i}\frac{d\alpha_i}{d\lambda_1} + \frac{\partial PI}{\partial(\lambda_1\beta_i)}\frac{d\lambda_1\beta_i}{d\lambda_1},$$

where

$$\frac{\partial PI}{\partial\alpha_i} = \frac{\lambda_1\beta_i\frac{\sigma_{xi}^2}{\sigma_i^2\alpha_i^2} + (\lambda_1\beta_i)^3\frac{\alpha_i^2-2\alpha_i+1}{2\alpha_i^4}}{\left(\sqrt{\frac{\sigma_{xi}^2}{\sigma_i^2} + \left[\lambda_1\beta_i\frac{\alpha_i-1}{\alpha_i}\right]^2 + \lambda_1^2\beta_i^2\frac{\alpha_i-1}{\alpha_i^2}}\right)^3} > 0$$

and

$$\frac{\partial PI}{\partial\lambda_1\beta_i} = \frac{\alpha_i - 1}{\alpha_i}\left[\frac{1}{2}\frac{1}{\sqrt{\frac{\sigma_{xi}^2}{\sigma_i^2} + \left[\lambda_1\beta_i\frac{\alpha_i-1}{\alpha_i}\right]^2 + \lambda_1^2\beta_i^2\frac{\alpha_i-1}{\alpha_i^2}}} + 2\frac{\sigma_{xi}^2}{\sigma_i^2(\lambda_1\beta_i)^3}\right] > 0.$$

Additionally, for $\lambda_1 < \lambda_L$, we have that $d\alpha_i/d\lambda_1 = 0$ and by Proposition 3, $\frac{d\lambda_1\beta_i}{d\lambda_1} > 0$, and so $dPI/d\lambda_1 > 0$. For large λ_1, we have

$$\lim_{\lambda_1 \to 1} \frac{\partial PI_i}{\partial\alpha_i} = 0,$$

with a bounded derivative $d\alpha_i/d\lambda_1$ and also $\frac{\partial PI}{\partial \lambda_1 \beta_i} > 0$ and $\frac{d\lambda_1 \beta_i}{d\lambda_1} < 0$ bounded away from zero, we have that $dPI_i/d\lambda_1 < 0$ for λ_1 sufficiently close to 1, but not necessarily equal to 1. □

A.10 Derivation of βs in the Duopoly Case

In this case the betas are:

$$\begin{aligned}
\beta_{0ji} &= 0 \\
\beta_{1ji} &= \frac{1}{\rho\hat{\sigma}_{ji}^2 + r\frac{dp_i}{dq_{ji}}}, \\
\beta_{2ji} &= \frac{1}{\rho\hat{\sigma}_{ji}^2 + r\frac{dp_i}{dq_{ji}}}, \\
\frac{dp_i}{dq_{ji}} &= \frac{\rho\sigma_i^2\lambda_j}{\lambda_0 r + \rho\sigma_i^2\lambda_k r\beta_{2ki}}.
\end{aligned}$$

so it is enough to find $\beta_{2ji} = \beta_{1ji}$. Plugging in:

$$\beta_{2ji} = \frac{1}{\rho\hat{\sigma}_{ji}^2 + \frac{\rho\sigma_i^2\lambda_j}{\lambda_0 + \rho\sigma_i^2\lambda_k\beta_{2ki}}}$$

which gives

$$\beta_{2ji}(\rho\hat{\sigma}_{ji}^2\lambda_0 + \rho\sigma_i^2\lambda_j) - \lambda_0 = \rho\sigma_i^2\lambda_k\beta_{2ki} - \beta_{2ji}\rho\hat{\sigma}_{ji}^2\rho\sigma_i^2\lambda_k\beta_{2ki}$$

and hence:

$$\frac{\beta_{2ji}(\rho\hat{\sigma}_{ji}^2\lambda_0 + \rho\sigma_i^2\lambda_j) - \lambda_0}{\lambda_k(\rho\sigma_i^2 - \beta_{2ji}\rho\hat{\sigma}_{ji}^2\rho\sigma_i^2)} = \beta_{2ki}$$

For an oligopolist k, analogous equations give:

$$\beta_{2ki}(\rho\hat{\sigma}_{ki}^2\lambda_0 + \rho\sigma_i^2\lambda_k) - \lambda_0 = \rho\sigma_i^2\lambda_j\beta_{2ji} - \beta_{2ki}\rho\hat{\sigma}_{ki}^2\rho\sigma_i^2\lambda_j\beta_{2ji}$$

plugging in

$$\begin{aligned}
&\frac{\beta_{2ji}(\rho\hat{\sigma}_{ji}^2\lambda_0 + \rho\sigma_i^2\lambda_j) - \lambda_0}{\lambda_k(\rho\sigma_i^2 - \beta_{2ji}\rho\hat{\sigma}_{ji}^2\rho\sigma_i^2)}(\rho\hat{\sigma}_{ki}^2\lambda_0 + \rho\sigma_i^2\lambda_k) - \lambda_0 = \\
&\rho\sigma_i^2\lambda_j\beta_{2ji} - \frac{\beta_{2ji}(\rho\hat{\sigma}_{ji}^2\lambda_0 + \rho\sigma_i^2\lambda_j) - \lambda_0}{\lambda_k(\rho\sigma_i^2 - \beta_{2ji}\rho\hat{\sigma}_{ji}^2\rho\sigma_i^2)}\rho\hat{\sigma}_{ki}^2\rho\sigma_i^2\lambda_j\beta_{2ji}
\end{aligned}$$

which is a quadratic equation:

$$\begin{aligned}
\beta_{2ji}^2 A + \beta_{2ji}B + C &= 0 \\
A &= \lambda_k\lambda_j\rho\sigma_i^2\rho\hat{\sigma}_{ji}^2\rho\sigma_i^2 + \rho\hat{\sigma}_{ki}^2\rho\sigma_i^2\lambda_j(\rho\hat{\sigma}_{ji}^2\lambda_0 + \rho\sigma_i^2\lambda_j) \\
B &= (\rho\hat{\sigma}_{ji}^2\lambda_0 + \rho\sigma_i^2\lambda_j)(\rho\hat{\sigma}_{ki}^2\lambda_0 + \rho\sigma_i^2\lambda_k) + \lambda_k\lambda_0\rho\hat{\sigma}_{ji}^2\rho\sigma_i^2 - \lambda_k\rho\sigma_i^2\rho\sigma_i^2\lambda_j \\
&\quad -\rho\hat{\sigma}_{ki}^2\rho\sigma_i^2\lambda_j\lambda_0 \\
C &= -\lambda_0(\lambda_k\rho\sigma_i^2 + \rho\hat{\sigma}_{ki}^2\lambda_0 + \rho\sigma_i^2\lambda_k)
\end{aligned}$$

There are two solutions for β_{2ji} and their product is negative; hence, only one positive solution—β_{2ji} is unique, and so is β_{2ki}. Collecting terms and simplifying gives:

$$\begin{aligned}
\beta_{2ji}^2 A + \beta_{2ji} B + C = 0 \\
A = \lambda_k \lambda_j \rho^3 \sigma_i^4 \hat{\sigma}_{ji}^2 + \rho^3 \hat{\sigma}_{ki}^2 \sigma_i^2 \lambda_j (\hat{\sigma}_{ji}^2 \lambda_0 + \sigma_i^2 \lambda_j) \\
B = \rho^2 \hat{\sigma}_{ji}^2 \hat{\sigma}_{ki}^2 \lambda_0^2 + 2\rho^2 \hat{\sigma}_{ji}^2 \sigma_i^2 \lambda_0 \lambda_k = \rho^2 \lambda_0 \hat{\sigma}_{ji}^2 (\hat{\sigma}_{ki}^2 \lambda_0 + 2\sigma_i^2 \lambda_k) \\
C = -\rho \lambda_0 (\hat{\sigma}_{ki}^2 \lambda_0 + 2\sigma_i^2 \lambda_k)
\end{aligned}$$

Or equivalently:

$$\beta_{2ji}^2 A + \beta_{2ji} B + C = 0 \tag{30}$$

$$A = \left[\lambda_k \lambda_j \rho^3 \sigma_i^4 \hat{\sigma}_{ji}^2 + \rho^3 \hat{\sigma}_{ki}^2 \sigma_i^2 \lambda_j (\hat{\sigma}_{ji}^2 \lambda_0 + \sigma_i^2 \lambda_j)\right] \frac{1}{(\hat{\sigma}_{ki}^2 \lambda_0 + 2\sigma_i^2 \lambda_k)} \tag{31}$$

$$B = \rho^2 \lambda_0 \hat{\sigma}_{ji}^2 \tag{32}$$

$$C = -\rho \lambda_0 \tag{33}$$

A.11 Proof of Proposition 4

. With the restriction that $\lambda_k = 1 - \lambda_0 - \lambda_j$, it is sufficient to show that A is increasing in λ_j.

$$\begin{aligned}
A &= \frac{(1-\lambda_0-\lambda_j)\lambda_j \rho^3 \sigma_i^4 \hat{\sigma}_{ji}^2 + \rho^3 \hat{\sigma}_{ki}^2 \sigma_i^2 \lambda_j (\hat{\sigma}_{ji}^2 \lambda_0 + \sigma_i^2 \lambda_j)}{(\hat{\sigma}_{ki}^2 \lambda_0 + 2\sigma_i^2 (1-\lambda_0-\lambda_j))} \\
B &= \rho^2 \lambda_0 \hat{\sigma}_{ji}^2 \\
C &= -\rho \lambda_0 \\
A &\propto \frac{(1-\lambda_0-\lambda_j)\lambda_j \sigma_i^2 \hat{\sigma}_{ji}^2 + \hat{\sigma}_{ki}^2 \lambda_j \left(\hat{\sigma}_{ji}^2 \lambda_0 + \sigma_i^2 \lambda_j\right)}{\hat{\sigma}_{ki}^2 \lambda_0 + 2\sigma_i^2 (1-\lambda_0-\lambda_j)} \\
\frac{\partial A}{\partial \lambda_j} &= \left[\left((1-\lambda_0-\lambda_j)\lambda_j \sigma_i^2 \hat{\sigma}_{ji}^2 + \hat{\sigma}_{ki}^2 \lambda_j \left(\hat{\sigma}_{ji}^2 \lambda_0 + \sigma_i^2 \lambda_j\right)\right)' \times \left(\hat{\sigma}_{ki}^2 \lambda_0 + 2\sigma_i^2 (1-\lambda_0-\lambda_j)\right)\right. \\
&\quad \left. - \left((1-\lambda_0-\lambda_j)\lambda_j \sigma_i^2 \hat{\sigma}_{ji}^2 + \hat{\sigma}_{ki}^2 \lambda_j \left(\hat{\sigma}_{ji}^2 \lambda_0 + \sigma_i^2 \lambda_j\right)\right) \times \left(\hat{\sigma}_{ki}^2 \lambda_0 + 2\sigma_i^2 (1-\lambda_0-\lambda_j)\right)'\right] \\
&\quad \times \frac{1}{\left(\hat{\sigma}_{ki}^2 \lambda_0 + 2\sigma_i^2 (1-\lambda_0-\lambda_j)\right)^2} \\
&\propto \left(\sigma_i^2 \hat{\sigma}_{ji}^2 \hat{\sigma}_{ki}^2 \lambda_0 (1-\lambda_0-2\lambda_j) + 2\sigma_i^4 \hat{\sigma}_{ji}^2 (1-\lambda_0-\lambda_j)(1-\lambda_0-2\lambda_j) + \hat{\sigma}_{ki}^4 \hat{\sigma}_{ji}^2 \lambda_0 \lambda_0\right. \\
&\quad \left. + 2\sigma_i^2 \hat{\sigma}_{ki}^2 \hat{\sigma}_{ji}^2 \lambda_0 (1-\lambda_0-\lambda_j) + 2\sigma_i^2 \hat{\sigma}_{ki}^2 \lambda_j \left(\hat{\sigma}_{ki}^2 \lambda_0 + 2\sigma_i^2 (1-\lambda_0-\lambda_j)\right)\right) \\
&\quad + 2(1-\lambda_0-\lambda_j)\lambda_j \sigma_i^4 \hat{\sigma}_{ji}^2 + 2\sigma_i^2 \hat{\sigma}_{ki}^2 \hat{\sigma}_{ji}^2 \lambda_0 \lambda_j + 2\sigma_i^4 \hat{\sigma}_{ki}^2 \lambda_j^2 \\
&= \sigma_i^2 \hat{\sigma}_{ji}^2 \hat{\sigma}_{ki}^2 \lambda_0 (3-3\lambda_0-2\lambda_j) + 2\sigma_i^4 \hat{\sigma}_{ji}^2 (1-\lambda_0-\lambda_j)^2 + \hat{\sigma}_{ki}^4 \hat{\sigma}_{ji}^2 \lambda_0 \lambda_0 \\
&\quad + 2\sigma_i^2 \hat{\sigma}_{ki}^4 \lambda_j \lambda_0 + 2\sigma_i^4 \hat{\sigma}_{ki}^2 \lambda_j (2-2\lambda_0-\lambda_j) \\
&> 0
\end{aligned}$$

Therefore $\dfrac{\partial \beta_{ji}}{\partial \lambda_j} < 0.$

□

A.12 Proof of Proposition 5

Part 1:

Price informativeness in the general model is:

$$PI_i = \frac{cov(p_i, z_i)}{\sigma_{pi}} = \frac{\sigma_i \sum_{j=1}^{l} \omega_{ji} \frac{\alpha_{ji}-1}{\alpha_{ji}}}{\sqrt{\frac{\sigma_{xi}^2}{\sigma_i^2} + \left[\sum_{j=1}^{l} \omega_{ji} \frac{\alpha_{ji}-1}{\alpha_{ji}}\right]^2 + \sum_{j=1}^{l} \omega_{ij}^2 \frac{\alpha_{ji}-1}{\alpha_{ji}^2}}}, \tag{34}$$

where

$$\omega_{ji} \equiv \frac{\partial \lambda_j q_{ji}}{\partial s_{ji}} = \lambda_j \beta_{1ji}$$

Dividing the numerator and the denominator by $\sum \omega_{ji}$ gives:

$$PI_i = \frac{\sigma_i \sum_{j=1}^{l} \frac{\omega_{ji}}{\sum \omega_{ji}} \frac{\alpha_{ji}-1}{\alpha_{ji}}}{\sqrt{\frac{\sigma_{xi}^2}{\sigma_i^2} \frac{1}{(\sum \omega_{ji})^2} + \left[\sum_{j=1}^{l} \frac{\omega_{ji}}{\sum \omega_{ji}} \frac{\alpha_{ji}-1}{\alpha_{ji}}\right]^2 + \sum_{j=1}^{l} (\frac{\omega_{ji}}{(\sum \omega_{ji}})^2 \frac{\alpha_{ji}-1}{\alpha_{ji}^2}}}, \tag{35}$$

For part 1 of the proposition, we consider a symmetric exogenous information allocation, denoted as α_i, which simplifies PI to:

$$PI_i = \frac{\sigma_i \frac{\alpha_i-1}{\alpha_i}}{\sqrt{\frac{\sigma_{xi}^2}{\sigma_i^2} \frac{1}{(\sum \omega_{ji})^2} + \left[\frac{\alpha_{ji}-1}{\alpha_{ji}}\right]^2 + \frac{\alpha_i-1}{\alpha_i^2} \sum_{j=1}^{l} (\frac{\omega_{ji}}{(\sum \omega_{ji}})^2}}, \tag{36}$$

Clearly, the last term in the denominator is minimized for $\lambda_j = \lambda_k$. The question remains on what is the behavior of the function

$$\Omega = \sum_j \omega_{ji} = \lambda_j \beta_{ji} + (1 - \lambda_0 - \lambda_j)\beta_{ki}$$

Given the expressions (31)-(33), we have:

$$\Omega_i = \frac{-(2(-1+\lambda_j+\lambda_0)\sigma_i^2 - \lambda_0\sigma_j^2)(\lambda_0\rho^2\sigma_j^2 - \sqrt{\frac{(\lambda_0\rho^4\sigma_j^2(2\lambda_j\sigma_i^2+\lambda_0\sigma_j^2)(-2(-1+\lambda_0)\sigma_i^2+\lambda_0\sigma_j^2))}{(-2(-1+\lambda_j+\lambda_0)\sigma_i^2+\lambda_0\sigma_j^2)}})}{(2\rho^3\sigma_i^2\sigma_j^2((-1+\lambda_0)\sigma_i^2 - \lambda_0\sigma_j^2))} + \frac{(2\lambda_j\sigma_i^2 + \lambda_0\sigma_j^2)(\lambda_0\rho^2\sigma_j^2 - \sqrt{\frac{(\lambda_0\rho^4\sigma_j^2(-2(-1+\lambda_0)\sigma_i^2+\lambda_0\sigma_j^2)(-2(-1+\lambda_j+\lambda_0)\sigma_i^2+\lambda_0\sigma_j^2))}{(2\lambda_j\sigma_i^2+\lambda_0\sigma_j^2)}})}{(2\rho^3\sigma_i^2\sigma_j^2((-1+\lambda_0)\sigma_i^2 - \lambda_0\sigma_j^2))}$$

The derivative of the expression with respect to λ_j is:

$$\frac{d\Omega_i}{d\lambda_j} = \frac{1}{(2\rho^3\sigma_i^2\sigma_{ji}^2((-1+\lambda_0)\sigma_i^2 - \lambda_0\sigma_{ji}^2))} \times$$
$$\frac{-2\sigma_i^2(\sigma_i^2 - \lambda_0\sigma_i^2 + \lambda_0\sigma_{ji}^2)\sqrt{\frac{(\lambda_0\rho^4\sigma_{ji}^2(2\lambda_j\sigma_i^2+\lambda_0\sigma_{ji}^2)(-2(-1+\lambda_0)\sigma_i^2+\lambda_0\sigma_{ji}^2))}{(-2(-1+\lambda_j+\lambda_0)\sigma_i^2+\lambda_0\sigma_{ji}^2)}}}{2\lambda_j\sigma_i^2 + \lambda_0\sigma_{ji}^2}$$
$$+\frac{2\sigma_i^2(\sigma_i^2 - \lambda_0\sigma_i^2 + \lambda_0\sigma_{ji}^2)\sqrt{\frac{(\lambda_0\rho^4\sigma_{ji}^2(-2(-1+\lambda_0)\sigma_i^2+\lambda_0\sigma_{ji}^2)(-2(-1+\lambda_j+\lambda_0)\sigma_i^2+\lambda_0\sigma_{ji}^2))}{(2\lambda_j\sigma_i^2+\lambda_0\sigma_{ji}^2)}})}{(-2(-1+\lambda_j+\lambda_0)\sigma_i^2 + \lambda_0\sigma_{ji}^2)}$$
$$-2\sigma_i^2(\lambda_0\rho^2\sigma_{ji}^2 - \sqrt{\frac{(\lambda_0\rho^4\sigma_{ji}^2(2\lambda_j\sigma_i^2+\lambda_0\sigma_{ji}^2)(-2(-1+\lambda_0)\sigma_i^2+\lambda_0\sigma_{ji}^2))}{(-2(-1+\lambda_j+\lambda_0)\sigma_i^2+\lambda_0\sigma_{ji}^2)}})$$
$$+2\sigma_i^2(\lambda_0\rho^2\sigma_{ji}^2 - \sqrt{\frac{(\lambda_0\rho^4\sigma_{ji}^2(-2(-1+\lambda_0)\sigma_i^2+\lambda_0\sigma_{ji}^2)(-2(-1+\lambda_j+\lambda_0)\sigma_i^2+\lambda_0\sigma_{ji}^2))}{(2\lambda_j\sigma_i^2+\lambda_0\sigma_{ji}^2)}})$$

In the above expression, the denominator (first expression) is always negative, so the sign of the derivative is negatively related to (after dividing the above by $2\sigma_i^4$ and simplifying):

$$\frac{d\Omega_i}{d\lambda_j} \propto \frac{(-1+2\lambda_j+\lambda_0)\sigma_i^2\sqrt{\frac{(\lambda_0\rho^4\sigma_{ji}^2(2\lambda_j\sigma_i^2+\lambda_0\sigma_{ji}^2)(-2(-1+\lambda_0)\sigma_i^2+\lambda_0\sigma_{ji}^2))}{(-2(-1+\lambda_j+\lambda_0)\sigma_i^2+\lambda_0\sigma_{ji}^2)}}}{(2\lambda_j\sigma_i^2+\lambda_0\sigma_{ji}^2)}$$
$$+\frac{((-1+2\lambda_j+\lambda_0)\sigma_i^2\sqrt{\frac{(\lambda_0\rho^4\sigma_{ji}^2(-2(-1+\lambda_0)\sigma_i^2+\lambda_0\sigma_{ji}^2)(-2(-1+\lambda_j+\lambda_0)\sigma_i^2+\lambda_0\sigma_{ji}^2))}{(2\lambda_j\sigma_i^2+\lambda_0\sigma_{ji}^2)}})}{(-2(-1+\lambda_j+\lambda_0)\sigma_i^2+\lambda_0\sigma_{ji}^2)}$$

Since both $(2\lambda_j\sigma_i^2 + \lambda_0\sigma_{ji}^2)$ and $(-2(-1+\lambda_j+\lambda_0)\sigma_i^2 + \lambda_0\sigma_{ji}^2)$ are strictly positive, the sign of the derivative depends negatively on the sign of $-1+2\lambda_j+\lambda_0$, which is negative for $\lambda_j < \frac{1-\lambda_0}{2}$, positive for $\lambda_j > \frac{1-\lambda_0}{2}$ and zero for $\lambda_j = \frac{1-\lambda_0}{2}$.
Hence, Ω_i is increasing for $\lambda_j < \frac{1-\lambda_0}{2}$, and decreasing for $\lambda_j > \frac{1-\lambda_0}{2}$, with a global maximum at $\lambda_j = \frac{1-\lambda_0}{2}$, which is equivalent to $\lambda_j = \lambda_k$.

Part 2:

The definition of PI when $\alpha_j \neq \alpha_i$ is:

$$PI = \frac{\sigma_i\left(\lambda_j\beta_{1ji}\frac{\alpha_{ji}-1}{\alpha_{ji}} + (1-\lambda_0-\lambda_j)\beta_{1ki}\frac{\alpha_{ki}-1}{\alpha_{ki}}\right)}{\sqrt{\frac{\sigma_{xi}^2}{\sigma_i^2} + \left[\lambda_j\beta_{1ji}\frac{\alpha_{ji}-1}{\alpha_{ji}} + (1-\lambda_0-\lambda_j)\beta_{1ki}\frac{\alpha_{ki}-1}{\alpha_{ki}}\right]^2 + \left(\lambda_j^2\beta_{1ji}^2\frac{\alpha_{ji}-1}{\alpha_{ji}^2} + (1-\lambda_0-\lambda_j)^2\beta_{1ki}^2\frac{\alpha_{ki}-1}{\alpha_{ki}^2}\right)}}$$

The first order condition is:

$$\sigma_i X = \frac{\left(\sigma_i\left(\lambda_j\beta_{1ji}\frac{\alpha_{ji}-1}{\alpha_{ji}} + (1-\lambda_0-\lambda_j)\beta_{1ki}\frac{\alpha_{ki}-1}{\alpha_{ki}}\right)\right)^3}{PI^2}\left(\left[\lambda_j\beta_{1ji}\frac{\alpha_{ji}-1}{\alpha_{ji}} + (1-\lambda_0-\lambda_j)\beta_{1ki}\frac{\alpha_{ki}-1}{\alpha_{ki}}\right]X\right.$$
$$\left. + \left(\left(\lambda_j\beta_{1ji}^2 + \lambda_j^2\beta_{1ji}\beta_{1ji}'\right)\frac{\alpha_{ji}-1}{\alpha_{ji}^2} + \left(-(1-\lambda_0-\lambda_j)\beta_{1ki}^2 + (1-\lambda_0-\lambda_j)^2\beta_{1ki}\beta_{1ki}'\right)\frac{\alpha_{ki}-1}{\alpha_{ki}^2}\right)\right)$$
$$X \equiv \left(\left(\beta_{1ji} + \lambda_j\beta_{1ji}'\right)\frac{\alpha_{ji}-1}{\alpha_{ji}} + \left(-\beta_{1ki} + (1-\lambda_0-\lambda_j)\beta_{1ki}'\right)\frac{\alpha_{ki}-1}{\alpha_{ki}}\right)$$

Where $\beta'_{1ki} = \frac{\partial \beta_{1ki}}{\partial \lambda_j}$ and $\beta'_{1ji} = \frac{\partial \beta_{1ji}}{\partial \lambda_j}$. If we set $\lambda_j = \lambda_k = 1 - \lambda_j - \lambda_0$, the expression can simplify to:

$$\begin{aligned}
0 &= \left(\beta_{1ji}\frac{\alpha_{ji}-1}{\alpha_{ji}} + \beta_{1ki}\frac{\alpha_{ki}-1}{\alpha_{ki}}\right)^3 \left(\left[\beta_{1ji}\frac{\alpha_{ji}-1}{\alpha_{ji}} + \beta_{1ki}\frac{\alpha_{ki}-1}{\alpha_{ki}}\right] X \right. \\
&\quad + \left(\beta_{1ji}\left(\beta_{1ji} + \lambda_j \beta'_{1ji}\right)\frac{\alpha_{ji}-1}{\alpha_{ji}^2} + \beta_{1ki}\left(-\beta_{1ki} + \lambda_j \beta'_{1ki}\right)\frac{\alpha_{ki}-1}{\alpha_{ki}^2} \right) \Bigg) \\
&\quad - X \frac{\left(\beta_{1ji}\frac{\alpha_{ji}-1}{\alpha_{ji}} + \beta_{1ki}\frac{\alpha_{ki}-1}{\alpha_{ki}}\right)^2}{\frac{\sigma_{xi}^2}{\sigma_i^2} + \lambda_j^2\left[\beta_{1ji}\frac{\alpha_{ji}-1}{\alpha_{ji}} + \beta_{1ki}\frac{\alpha_{ki}-1}{\alpha_{ki}}\right]^2 + \lambda_j^2\left(\beta_{1ji}^2\frac{\alpha_{ji}-1}{\alpha_{ji}^2} + \beta_{1ki}^2\frac{\alpha_{ki}-1}{\alpha_{ki}^2}\right)} \\
0 &= \left(\beta_{1ji}\frac{\alpha_{ji}-1}{\alpha_{ji}} + \beta_{1ki}\frac{\alpha_{ki}-1}{\alpha_{ki}}\right) \left(\left[\beta_{1ji}\frac{\alpha_{ji}-1}{\alpha_{ji}} + \beta_{1ki}\frac{\alpha_{ki}-1}{\alpha_{ki}}\right] \right. \\
&\quad \left(\left(\beta_{1ji} + \lambda_j \beta'_{1ji}\right)\frac{\alpha_{ji}-1}{\alpha_{ji}} + \left(-\beta_{1ki} + \lambda_j \beta'_{1ki}\right)\frac{\alpha_{ki}-1}{\alpha_{ki}} \right) \\
&\quad + \left(\beta_{1ji}\left(\beta_{1ji} + \lambda_j \beta'_{1ji}\right)\frac{\alpha_{ji}-1}{\alpha_{ji}^2} + \beta_{1ki}\left(-\beta_{1ki} + \lambda_j \beta'_{1ki}\right)\frac{\alpha_{ki}-1}{\alpha_{ki}^2} \right) \Bigg) \\
&\quad - \frac{\left(\left(\beta_{1ji} + \lambda_j \beta'_{1ji}\right)\frac{\alpha_{ji}-1}{\alpha_{ji}} + \left(-\beta_{1ki} + \lambda_j \beta'_{1ki}\right)\frac{\alpha_{ki}-1}{\alpha_{ki}} \right)}{\frac{\sigma_{xi}^2}{\sigma_i^2} + \lambda_j^2\left[\beta_{1ji}\frac{\alpha_{ji}-1}{\alpha_{ji}} + \beta_{1ki}\frac{\alpha_{ki}-1}{\alpha_{ki}}\right]^2 + \lambda_j^2\left(\beta_{1ji}^2\frac{\alpha_{ji}-1}{\alpha_{ji}^2} + \beta_{1ki}^2\frac{\alpha_{ki}-1}{\alpha_{ki}^2}\right)} \\
&= F(\alpha_{ji}, \alpha_{ki}, \lambda_0, \lambda_j, \cdot) - G(\alpha_{ji}, \alpha_{ki}, \lambda_0, \lambda_j, \cdot, \sigma_{xi})
\end{aligned}$$

The function G in this expression is a function of σ_{xi}^2, while the function F is not. Therefore, while there could be a set of parameter values that satisfy the above expression, in order for there to be a general solution, it must be that the case that both F and G are equal to zero, as they will not be equal to each other outside of a measure-0 set of values for σ_{xi}^2. Therefore, the conditions to be satisfied become:

$$\begin{aligned}
0 &= \left(\beta_{1ji} + \lambda_j \beta'_{1ji}\right)\frac{\alpha_{ji}-1}{\alpha_{ji}} + \left(-\beta_{1ki} + \lambda_j \beta'_{1ki}\right)\frac{\alpha_{ki}-1}{\alpha_{ki}} \\
0 &= \left(\beta_{1ji}\frac{\alpha_{ji}-1}{\alpha_{ji}} + \beta_{1ki}\frac{\alpha_{ki}-1}{\alpha_{ki}}\right) \left(\left[\beta_{1ji}\frac{\alpha_{ji}-1}{\alpha_{ji}} + \beta_{1ki}\frac{\alpha_{ki}-1}{\alpha_{ki}}\right] \right. \\
&\quad \left(\left(\beta_{1ji} + \lambda_j \beta'_{1ji}\right)\frac{\alpha_{ji}-1}{\alpha_{ji}} + \left(-\beta_{1ki} + \lambda_j \beta'_{1ki}\right)\frac{\alpha_{ki}-1}{\alpha_{ki}} \right) \\
&\quad + \left(\beta_{1ji}\left(\beta_{1ji} + \lambda_j \beta'_{1ji}\right)\frac{\alpha_{ji}-1}{\alpha_{ji}^2} + \beta_{1ki}\left(-\beta_{1ki} + \lambda_j \beta'_{1ki}\right)\frac{\alpha_{ki}-1}{\alpha_{ki}^2} \right) \Bigg)
\end{aligned}$$

$\left(\beta_{1ji}\frac{\alpha_{ji}-1}{\alpha_{ji}} + \beta_{1ki}\frac{\alpha_{ki}-1}{\alpha_{ki}}\right)$ is strictly positive, so the two conditions are :

$$\begin{aligned}
0 &= \left(\beta_{1ji} + \lambda_j \beta'_{1ji}\right)\frac{\alpha_{ji}-1}{\alpha_{ji}} + \left(-\beta_{1ki} + \lambda_j \beta'_{1ki}\right)\frac{\alpha_{ki}-1}{\alpha_{ki}} \\
0 &= \left(\beta_{1ji}\left(\beta_{1ji} + \lambda_j \beta'_{1ji}\right)\frac{\alpha_{ji}-1}{\alpha_{ji}^2} + \beta_{1ki}\left(-\beta_{1ki} + \lambda_j \beta'_{1ki}\right)\frac{\alpha_{ki}-1}{\alpha_{ki}^2} \right) \Bigg)
\end{aligned}$$

Suppose that we assume that the first condition is always satisfied. Then we can rewrite the second condition as:

$$\frac{\beta_{ji}}{\alpha_{ji}} = \frac{\beta_{ki}}{\alpha_{ki}}$$

Rearranging gives the necessary condition for symmetric λs to maximize PI for asymmetric learning:

$$\frac{\beta_{ji}}{\beta_{ki}} = \frac{\alpha_{ji}}{\alpha_{ki}}$$

Consider the left hand size of the above expression. Plugging in the solution using expressions (31)-(33) for duopolist j and an analogous set of equations for oligopolist k gives:

$$\frac{\beta_{ji}}{\beta_{ki}} = \frac{((1-\lambda_0)\sigma_i^2 + \lambda_0\hat{\sigma}_{ji}^2)(\lambda_0\rho^2\hat{\sigma}_{ki}^2 - \sqrt{\frac{\lambda_0\rho^4((1-\lambda_0)\sigma_i^2+\lambda_0\hat{\sigma}_{ki}^2)(\lambda_0\hat{\sigma}_{ji}^2\hat{\sigma}_{ki}^2+(1-\lambda_0)\sigma_i^2(\hat{\sigma}_{ji}^2+\hat{\sigma}_{ki}^2))}{(1-\lambda_0)\sigma_i^2+\lambda_0\hat{\sigma}_{ji}^2}})}{((1-\lambda_0)\sigma_i^2 + \lambda_0\hat{\sigma}_{ki}^2)(\lambda_0\rho^2\hat{\sigma}_{ji}^2 - \sqrt{\frac{\lambda_0\rho^4((1-\lambda_0)\sigma_i^2+\lambda_0\hat{\sigma}_{ji}^2)(\lambda_0\hat{\sigma}_{ji}^2\hat{\sigma}_{ki}^2+(1-\lambda_0)\sigma_i^2(\hat{\sigma}_{ji}^2+\hat{\sigma}_{ki}^2))}{(1-\lambda_0)\sigma_i^2+\lambda_0\hat{\sigma}_{ki}^2}})} \tag{37}$$

Consider first the numerator of this expression. Multiplying through by the first expression:

$$((1-\lambda_0)\sigma_i^2 + \lambda_0\hat{\sigma}_{ji}^2)\lambda_0\rho^2\hat{\sigma}_{ki}^2 - \sqrt{\lambda_0\rho^4((1-\lambda_0)\sigma_i^2 + \lambda_0\hat{\sigma}_{ji}^2)((1-\lambda_0)\sigma_i^2 + \lambda_0\hat{\sigma}_{ki}^2)(\lambda_0\hat{\sigma}_{ji}^2\hat{\sigma}_{ki}^2 + (1-\lambda_0)\sigma_i^2(\hat{\sigma}_{ji}^2 + \hat{\sigma}_{ki}^2))}$$

Factoring out ρ^2 and λ_0 and rearranging gives:

$$\lambda_0(1-\lambda_0)\alpha_{ji}\hat{\sigma}_{ji}^2\hat{\sigma}_{ki}^2 + \lambda_0^2\hat{\sigma}_{ji}^2\hat{\sigma}_{ki}^2 - \lambda_0^2\sqrt{(\frac{1-\lambda_0}{\lambda_0}\sigma_i^2 + \hat{\sigma}_{ji}^2)(\frac{1-\lambda_0}{\lambda_0}\sigma_i^2 + \hat{\sigma}_{ki}^2)(\hat{\sigma}_{ji}^2\hat{\sigma}_{ki}^2 + \frac{1-\lambda_0}{\lambda_0}\sigma_i^2(\hat{\sigma}_{ji}^2 + \hat{\sigma}_{ki}^2))} \tag{38}$$

Performing analogous calculations for the denominator of (37), we get

$$\lambda_0(1-\lambda_0)\alpha_{ki}\hat{\sigma}_{ji}^2\hat{\sigma}_{ki}^2 + \lambda_0^2\hat{\sigma}_{ji}^2\hat{\sigma}_{ki}^2 - \lambda_0^2\sqrt{(\frac{1-\lambda_0}{\lambda_0}\sigma_i^2 + \hat{\sigma}_{ji}^2)(\frac{1-\lambda_0}{\lambda_0}\sigma_i^2 + \hat{\sigma}_{ki}^2)(\hat{\sigma}_{ji}^2\hat{\sigma}_{ki}^2 + \frac{1-\lambda_0}{\lambda_0}\sigma_i^2(\hat{\sigma}_{ji}^2 + \hat{\sigma}_{ki}^2))} \tag{39}$$

For the ratio of (38) and (39) to be equal to $\frac{\alpha_{ji}}{\alpha_{ki}}$ in the case of $\alpha_{ji} \neq \alpha_{ki}$, it has to be that

$$\lambda_0^2\hat{\sigma}_{ji}^2\hat{\sigma}_{ki}^2 - \lambda_0^2\sqrt{(\frac{1-\lambda_0}{\lambda_0}\sigma_i^2 + \hat{\sigma}_{ji}^2)(\frac{1-\lambda_0}{\lambda_0}\sigma_i^2 + \hat{\sigma}_{ki}^2)(\hat{\sigma}_{ji}^2\hat{\sigma}_{ki}^2 + \frac{1-\lambda_0}{\lambda_0}\sigma_i^2(\hat{\sigma}_{ji}^2 + \hat{\sigma}_{ki}^2))} = 0$$

Factoring out $\lambda_0^2\hat{\sigma}_{ji}^2\hat{\sigma}_{ki}^2$, this condition boils down to

$$(\frac{1-\lambda_0}{\lambda_0}\alpha_{ji} + 1)(\frac{1-\lambda_0}{\lambda_0}\alpha_{ki} + 1)(1 + \frac{1-\lambda_0}{\lambda_0}\sigma_i^2\frac{\hat{\sigma}_{ji}^2 + \hat{\sigma}_{ki}^2}{\hat{\sigma}_{ji}^2\hat{\sigma}_{ki}^2}) = 1$$

which is never satisfied.

Part 3:

For Part 3, observe that for the case of $\alpha_{ji} = 1$ and $\alpha_{ki} > 1$, we have:

$$PI_i = \frac{\sigma_i\frac{\alpha_{ki}-1}{\alpha_{ki}}}{\sqrt{\frac{\sigma_{xi}^2}{\sigma_i^2}\frac{1}{\omega_{ki}^2} + \left[\frac{\alpha_{ji}-1}{\alpha_{ji}}\right]^2 + \frac{\alpha_{ki}-1}{\alpha_{ki}^2}}}, \tag{40}$$

The question becomes which value of λ_j maximizes information pass-through ω_{ki}, where

$$\omega_{ki} = \lambda_k\beta_{ki} = (1-\lambda_0-\lambda_j)\beta_{ki},$$

and β_{ki} is determined by (31)-(33). In particular, the sign of the derivative of ω_{ki} with respect to λ_j depends on the sign of:

$$\hat{\sigma}^2_{ki}((-2+\lambda_0)\sigma_i^2 - \lambda_0\hat{\sigma}^2_{ki})$$
$$+[-\rho^2 4(-1+\lambda_j+\lambda_0)^3\sigma_i^6 + 2(-1+\lambda_j+\lambda_0)(4\lambda_j^2+\lambda_0(-3+4\lambda_0)+\lambda_j(-2+8\lambda_0))\sigma_i^4\hat{\sigma}^2_{ki}$$
$$-(4\lambda_j^3+6\lambda_j(-1+2\lambda_j)\lambda_0+2(-2+7\lambda_j)\lambda_0^2+5\lambda_0^3)\sigma_i^2\hat{\sigma}^4_{ki}+\lambda_0^2(2\lambda_j+\lambda_0)\hat{\sigma}^6_{ki}]\times$$
$$\frac{1}{(2\lambda_j+\lambda_0)\sqrt{\frac{(\lambda_0\rho^4(4(-1+\lambda_j+\lambda_0)^2\sigma_i^4-4(-1+\lambda_j+\lambda_0)(\lambda_j+\lambda_0)\sigma_i^2\hat{\sigma}^2_{ki}+\lambda_0(2\lambda_j+\lambda_0)\hat{\sigma}^4_{ki}))}{(2\lambda_j+\lambda_0)}}}$$

Consider first the limit case of $\lambda_j = 0$, the above function converges to

$$-4(-1+\lambda_0)^3\sigma_i^6 + 2(-1+\lambda_0)\lambda_0(-3+4\lambda_0)\sigma_i^4\sigma^2_{ki} + (4-5\lambda_0)\lambda_0^2\sigma_i^2\sigma^4_{ki} + \lambda_0^3\sigma^6_{ki} \rightarrow_{\lambda_0\rightarrow 0} = 4\sigma_i^6 > 0$$

so that for small enough λ_0, it is always beneficial from the PI perspective to increase the size of the passive oligopolist. The reason for that is that the information pass-through of the active oligopolist is decreasing enough with their size that it actually reducing their size increases PI. The conclusion is that if the fringe sector is small enough, an interior solution for λ_j maximizes PI.

A.13 Proof of Proposition 6

Proof. It is sufficient to show that $\frac{d\beta_{ji}}{d\hat{\sigma}^2_{ki}} < 0$, which is true if and only if $\frac{\partial A}{\partial \hat{\sigma}^2_{ki}} > 0$. Using (31):

$$\frac{\partial A}{\partial \hat{\sigma}^2_{ki}} = -\frac{(\lambda_j(-1+\lambda_j+\lambda_0)\sigma_i^4(2\lambda_j\sigma_i^2+\lambda_0\sigma^2_{ji})\rho^3)}{(-2(-1+\lambda_j+\lambda_0)\sigma_i^2+\lambda_0\sigma_k^2)^2}$$

which is equal in sign to

$$-(-1+\lambda_j+\lambda_0) > 0.$$

□

A.14 Existence Proofs

Perfect Competition

Existence of a unique symmetric trading equilibrium is established following Admati (1985).

Existence of a unique learning equilibrium follows the waterfilling argument of Kacperczyk et al (2016)

Monopoly

Existence of a unique trading equilibrium follows from the unique solution to the trading problem:

$$U_j = \max_{\{q_j\}_{i=1}^n} E[W_j] - \frac{rho}{2}V[W_j] \quad s.t. \quad W_j = (1+r)W_0 + \sum_{i=1}^n q_{ji}(z_i - rp_i)$$

which is

$$q_{ji} = \frac{\hat{\mu}_{ji} - rp_i}{\rho\hat{\sigma}^2_{ji} + r\frac{dp_i}{dq_{ji}}}$$

For the learning decision, the monopolist's problem is:

$$U = \frac{1}{2\rho}\sum_{i=1}^n \frac{\frac{L_i}{\lambda_0}\alpha_i + \lambda_0(\alpha_i - 1)}{\lambda_0 + 2\lambda_1\alpha_i}$$

where $L_i \equiv \rho^2(\bar{x}_i^2 + \sigma_{xi}^2)\sigma_i^2$. The first derivative is:

$$\frac{\partial U}{\partial \alpha_i} = \frac{L_i + 1 - \lambda_1^2}{(\lambda_0 + 2\lambda_1\alpha_i)^2}$$

Because the learning constraint is a capacity constraint, the first order condition is:

$$\frac{L_2 + 1 - \lambda_1^2}{(\lambda_0 + 2\lambda_1\alpha_i)^2}\alpha_i \leq \theta$$

where θ is the Lagrange multiplier on the capacity constraint. The second order condition to establish an internal solution is:

$$\frac{(L_i + 1 - \lambda_1^2)(\lambda_0 - 2\lambda_1\alpha_i)}{(\lambda_0 + 2\lambda_1\alpha_i)^3} < 0$$

This is satisfied whenever $\lambda_1 > \frac{1}{3}$. Therefore, there is an interior (unique) learning solution whenever $\lambda_1 > \frac{1}{3}$. However, when $\lambda_1 \leq \frac{1}{3}$, the learning solution is a corner, which means the monopolist will devote all their learning capacity to the asset with the largest L_i.

Duopoly

We have taken learning as exogenously specified in this setting, so it is sufficient to show that the trading equilibrium is unique. Because we have linear trading strategies, we must show that the coefficients on those trading strategies have a unique solutiuon. Because we have also assumed that there is no learning from prices, we have that $\beta_{1ji} = \beta_{2ji}$ and that $\beta_{0ji} = 0$. The functional form of β_{ji} is:

$$\beta_{ji} = \frac{1}{\rho\hat{\sigma}_{ji^2} + r\frac{dp_i}{dq_{ji}}}$$

and substituting for the price impact term $\frac{dp_i}{dq_{ji}} = \frac{\rho\sigma_i^2\lambda_j}{\lambda_0 + \rho\sigma_i^2\lambda_k\beta_{ki}}$ we get that:

$$\beta_{ki} = \frac{\beta_{ji}(\rho\hat{\sigma}_{ji}^2\lambda_0 + \rho\sigma_i^2\lambda_j) - \lambda_0}{\lambda_k(\rho\sigma_i^2 - \beta_{ji}\rho\hat{\sigma}_{ji}^2\rho\sigma_i^2}$$

and a symmetric equation for β_{ji} as a function of β_{ki}. Substituting, we get that β_{ji} is the solution to the following quadratic:

$$A\beta_{ji}^2 + B\beta_{ji} + C = 0$$

where

$$\begin{aligned} A &= \lambda_k\lambda_j\rho^3\sigma_i^4\hat{\sigma}_{ji}^2 + \rho^3\sigma_i^2\hat{\sigma}_{ki}^2\lambda_j(\hat{\sigma}_{ji}^2\lambda_0 + \sigma_i^2\lambda_j) \\ B &= \rho^2(\hat{\sigma}_{ji}^2\lambda_0 + \sigma_i^2\lambda_j)(\hat{\sigma}_{ki}^2\lambda_0 + \sigma_i^2\lambda_k) + \lambda_k\lambda_0\rho\hat{\sigma}_{ji}^2\sigma_i^2 - \rho^2\sigma_i^4\lambda_k\lambda_j - \rho^2\hat{\sigma}_{ki}^2\sigma_i^2\lambda_j\lambda_0 \\ C &= -\lambda_0\rho(2\lambda_k\sigma_i^2 + \hat{\sigma}_{ki}^2\lambda_0) \end{aligned}$$

A is positive and C is negative, which means there are two real roots, with one positive and one negative. Therefore, there is a unique solution for β_{ji} and a unique trading equilibrium.